CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

9794 MATHEMATICS

9794/02 Paper 2 (Pure Mathematics 2), maximum raw mark 80

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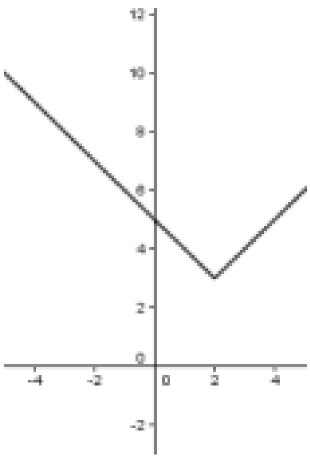
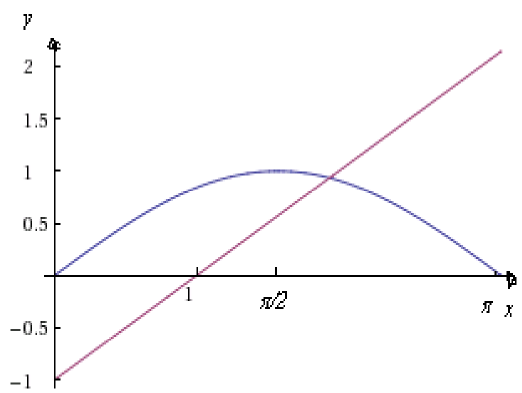
International Examinations

1	$\frac{31}{6-\sqrt{5}} \times \frac{6+\sqrt{5}}{6+\sqrt{5}} = \frac{186+31\sqrt{5}}{31}$ $= 6+\sqrt{5}$	M1 B1 A1 [3]	Show intention to multiply top and bottom by $6+\sqrt{5}$ Correct denominator; at least as far as 36 – 5 Show given answer correctly, including 31 seen as denominator before cancelling If showing that $\left(6+\sqrt{5}\right)\left(6-\sqrt{5}\right)=31$ then M1 – attempt to expand A1 – at least 36 – 5 seen A1 – obtain 31
2	$\int 6x^2 + 2\mathrm{d}x = 2x^3 + 2x\left(+c\right)$ $3 = 2 + 2 + c$ so $c = -1$ $y = 2x^3 + 2x - 1$	M1* A1 M1d* A1 [4]	Attempt to integrate at least one term – increase in power by 1 Obtain correct integral (allow no $+c$) Substitute (1, 3) to find c Correct equation, including $y =$
3 (i)		M1 A1 A1 [3]	Sketch V-shape graph, vertex in any quadrant Vertex at (2, 3), y-intercept at 5 Fully correct graph for at least $-5 \le x \le 5$
(ii)	Two x-values correspond to the same y-value	B1 [1]	Or give numerical example such as f(1) = 4 = f(3) Referring to just 'multiple' or 'many' x-values is B0 Must be using correct f(x), so not just $\lvert x-2 \rvert$ B0 if additional incorrect statement, such as 'many to one'

4	$\pi\int_1^2 x^6\,dx = \pi\left[\frac{x^7}{7}\right]_1^2$ $= \pi\left(\frac{128}{7} - \frac{1}{7}\right) = \frac{127}{7}\pi$ (= 57.0 to 3 sf)	B1 M1 M1 A1 [4]	State or imply correct formula for volume of revolution Attempt integration to obtain kx^7 Attempt use of limits in any integration attempt (i.e. increase in power by 1) Must be correct order and subtraction (M0M1 is possible) Obtain $\frac{127\pi}{7}$, or 57.0 or better (allow $\pi\frac{127}{7}$)
5 (i)	f(1.5) = 0.497494... f(2) = –0.090702...	M1 A1 [2]	Attempt evaluation of f(1.5) and f(2) – evaluation must be seen so f(1.5) > 0 is not sufficient Conclude correctly – refer to sign change oe Must have correct values for f(1.5) and f(2) Allow rounded or truncated values – 1sf, or better
(ii)	e.g., starting with $x_0 = 1.5$ $x_1 = 1.9974\ldots$ $x_2 = 1.9103\ldots$ $x_3 = 1.9429\ldots$ $x = 1.93$ to 2 dp	B1 M1 A1 [3]	Correct first iterate – must start with $1.5 \le x \le 2$ f(1.75) = 1.9839... , f(2) = 1.9092... Correct iteration process (at least 3) Allow iteration in degrees (gives 1.0177...) Obtain 1.93 – must be 2dp exactly Must be clear conclusion for root so A0 for e.g. $x_6 = 1.93$
(iii)		M1* A1 [2]	Sketch attempt at sine graph, with period of 2π, and a positive linear graph, with negative y-intercept Both graphs fully correct for [0, π], with some indication of scale on both axes and with], with some indication of scale on both axes and with $y = x - 1$ passing though ($\frac{\pi}{2}$, ≈ 0.6)
(iv)	One point of intersection oe	B1 d* [1]	Allow 'they will not cross again' or equivalent

6	(i)	$\frac{dT}{dt}$ is the rate of change of T $T-20$ is difference between T and the temp of the room. k is the constant of proportionality negative since the temperature is decreasing.	B1 B1 [2]	At least two correct points Fully correct explanation
	(ii)	$\int \frac{1}{T-20}\,dT = \int -k\,dt$ $\ln\lvert T-20\rvert$ $= -kt + c$ $\ln 60 = c$ $T-20 = e^{-kt+\ln 60} = e^{-kt}e^{\ln 60} = 60e^{-kt}$ $T = 20 + 60e^{-kt}$	M1 A1 A1 M1 M1 A1 [6]	Separate variables and attempt integration of both sides oe Correct $\ln(T-20)$ Correct $-kt$ (allow no $+c$) Attempt c using $T = 80$, $t = 0$ (clear detail required if rearrangement is done first e.g. $80 = 20 + A$) Could be using any function, following clear integration attempt Rearrange expression of form $\pm \ln\lvert T-20\rvert = \pm kt \pm c$ to given expression, including correct manipulation of logs and exponentials – allow if still in terms of c Must be sound algebra throughout Obtain $T = 20 + 60e^{-kt}$, detail required and no errors seen Must see $e^{-kt+\ln 60} = e^{-kt}e^{\ln 60} = 60e^{-kt}$ (oe in terms of c)
	(iii)	$\ln 40 = -2k + \ln 60$ OR $60 = 20 + 60e^{-2k}$ $2k = \ln\frac{3}{2}$ $\quad e^{-2k} = \frac{2}{3}$ $k = \frac{1}{2}\ln\frac{3}{2}$ $\quad k = -\frac{1}{2}\ln\frac{2}{3}$	M1 M1 A1 [3]	Substitute $T = 60$, $t = 2$ into given expression, oe Attempt to find k, allow one slip but must be using correct order of operations Obtain $k = \frac{1}{2}\ln\frac{3}{2}$ oe, including 0.203

7 (i)	$x^3 = 27t^3$ $y = 1 + \frac{1}{27}x^3$ AG	M1 A1 [2]	Attempt to eliminate t Obtain given answer convincingly M1A0 for $y = 1 + \left(\frac{x}{3}\right)^3 = 1 + \frac{1}{27}x^3$
(ii)	$1 + \frac{1}{27}x^3 = x^2 + 4x - 19$ $x^3 - 27x^2 - 108x + 540 = 0$ $(x-3)(x^2 - 24x - 180) = 0$ $(x-30)(x+6) = 0$ $x = 30$ or -6 points $(30, 1001)$ and $(-6, -7)$ OR $1 + t^3 = 9t^2 + 12t - 19$ $t^3 - 9t^2 - 12t + 20 = 0$ $(t-1)(t^2 - 8t - 20) = 0$ $(t-1)(t-10)(t+2) = 0$ $t = 1, 10$ or -2 points $(30, 1001)$ and $(-6, -7)$	M1 M1* A1 M1d* A1 A1 [6] M1 M1* A1 M1d* A1 A1	Reduce to equation in one variable Attempt division by $(x-3)$ Obtain correct quotient Attempt to solve quadratic quotient Obtain correct roots Obtain coordinates of both points Reduce to equation in one variable Attempt division by $(t-1)$ Obtain correct quotient Attempt to solve quadratic quotient Correct factorisation (could be implied by roots) Obtain coordinates of both points
8	$f'(x) = \frac{2x(3x^2-1) - x^2(6x)}{(3x^2-1)^2}$ $= \frac{-2x}{(3x^2-1)^2}$ for $x > 0$, $-2x < 0$ and $(\ldots)^2 > 0$ and $\frac{-\text{ve}}{+\text{ve}} < 0$ hence decreasing function	M1 A1 A1 M1 A1 [5]	Attempt use of quotient rule, or equivalent Correct unsimplified expression Correct simplified expression Identify that $f'(x) < 0$ is required; allow 'gradient' for $f'(x)$ Show convincingly that the denominator is always positive and the numerator is always negative for $x > 1$, and hence $f'(x) < 0$ Graphical solutions could get M1 for $f'(x) < 0$ is required, but need to show no stationary points to get any further credit

9	$2y\dfrac{dy}{dx} = 4x^3 - 12x^2$	M1 A1	Differentiate implicitly to get at least LHS Obtain fully correct expression
	$4x^2(x-3) = 0$	B1	Use $\dfrac{dy}{dx} = 0$
		M1	Attempt to solve for x
	$x = 0$ or $x = 3$	A1	Obtain $x = 0, 3$, www
	$x = 0 \rightarrow y^2 = 36 \rightarrow y = \pm 6$ $x = 3 \rightarrow y^2 = 9 \rightarrow y = \pm 3$	M1	Attempt to find y, must include square rooting
	hence equations are	A1	Obtain at least two correct equations, www
	$y = 3,\ y = -3,\ y = 6,\ y = -6$	A1	Obtain all four correct equations, and no others (A1 A0 if final eqns given as $y = \pm 3$, $y = \pm 6$) Misreading y for y^2 gets M0A0B1M1A1M1A1A0 Using $y = \sqrt{x^4 - 4x^3 + 36}$ can get full marks
		[8]	
10 (i)	$\sin\left(2\theta + \dfrac{1}{2}\pi\right) = \sin 2\theta \cos\dfrac{1}{2}\pi + \sin\dfrac{1}{2}\pi \cos 2\theta$	M1	Use correct expansion
	$\cos\dfrac{1}{2}\pi = 0, \quad \sin\dfrac{1}{2}\pi = 1$ so		These values must be explicit or implied in method for A1
	$\sin\left(2\theta + \dfrac{1}{2}\pi\right) = \cos 2\theta$	A1	Obtain given answer convincingly
		[2]	Also allow arguments by linear transformations
(ii)	$\sin\left(2\theta + \dfrac{1}{2}\pi\right) = \sin 3\theta$,		
	A: $2\theta + \dfrac{1}{2}\pi = 3\theta \Rightarrow \theta = \dfrac{1}{2}\pi$	B1	Obtain $\dfrac{1}{2}\pi$
	B: $3\theta = \pi - \left(2\theta + \dfrac{1}{2}\pi\right)$	M1	Attempt second solution using symmetry of sin curve oe
	$\theta = \dfrac{1}{10}\pi$	A1	Obtain $\dfrac{1}{10}\pi$
	$3\theta = \pi - \left(2\theta + \dfrac{1}{2}\pi\right) + 4\pi \Rightarrow \theta = \dfrac{9}{10}\pi$	A1	Obtain $\dfrac{9}{10}\pi$
	$3\theta = \pi - \left(2\theta + \dfrac{1}{2}\pi\right) + 6\pi \Rightarrow \theta = \dfrac{13}{10}\pi$	A1	Obtain $\dfrac{13}{10}\pi$
	$3\theta = \pi - \left(2\theta + \dfrac{1}{2}\pi\right) + 8\pi \Rightarrow \theta = \dfrac{17}{10}\pi$	A1	Obtain $\dfrac{17}{10}\pi$ Accept decimal equivalents for each root After B1M1A1 given, apply penalty of –1 against final three A marks for each additional incorrect root
		[6]	

(iii)	$\sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$	M1*	Expand using $\sin(2\theta + \theta)$ Or use De Moivre's theorem
	$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$	M1d*	Attempt to get expression in terms of $\sin\theta$ only
	$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ $= 3\sin\theta - 4\sin^3\theta$	A1	Obtain given answer convincingly
	$\cos 2\theta - \sin 3\theta$ (= 0) $(1 - 2\sin^2\theta) - (3\sin\theta - 4\sin^3\theta)$ (= 0) $4\sin^3\theta - 2\sin^2\theta - 3\sin\theta + 1$ (= 0)	 M1 M1	 Attempt to rearrange to comparable format Identify $x = \sin\theta$ (could be implied) and attempt to use solution(s) from part **(ii)**
	$x = 0.309$, 1 or -0.809 to 3sf	A1 [6]	Obtain $x = 0.309$, 1, -0.809 (allow 2dp) Allow surd values of $\frac{1}{4}\left(-1\pm\sqrt{5}\right)$
11 (i)	$RS = r\theta$ $RT = r\tan\theta$ $OT = r\sec\theta$ $ST = r\sec\theta - r$ $P = r\sec\theta - r + r\theta + r\tan\theta$	B1 B1 M1 A1	Correct RS Correct RT Attempt ST Fully correct expression for P (could be $\frac{1}{\cos\theta}$ for $\sec\theta$, but not $\sqrt{1+\tan^2\theta}$
	$A = \frac{1}{2}RT \times OR$	M1	Attempt area of triangle – must be valid attempt at RT
	$-\frac{1}{2}r^2\theta$	B1	State correct area of sector
	$= \frac{1}{2}r^2\left(\tan\theta - \theta\right)$	A1 [7]	Correct expression for A
(ii)	Let $A = rP$,	M1	Equate A with rP (allow use of $\neq$)
	$r^2\sec\theta - r^2 + r^2\theta + r^2\tan\theta = \frac{1}{2}r^2\left(\tan\theta - \theta\right)$		
	so $2\sec\theta - 2 + \tan\theta + 3\theta = 0$	M1	Attempt to justify why no solutions – correct equation only
	for $0 < \theta < \frac{1}{2}\pi$, $\sec\theta > 1$, so $2\sec\theta - 2 > 0$.	B1	State $\sec\theta > 1$, or equivalent
	Since $\tan\theta > 0$ and $\theta > 0$ equality is impossible, so we have a contradiction.	A1 [4]	Fully correct and convincing argument