Cambridge Assessment International Education

Cambridge Pre-U

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **October/November 2020**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Partial Marks	Guidance
1	$n = 28$	**B1**	Any or no method
	$\frac{7+196}{2}\times 28$ or $\frac{28}{2}(2\times 7+27\times 7)$	**M1**	Correct formula with $a = 7$ (or $a = 0$), $d = 7$, but allow *their n* Possibly implied by correct answer
	2842	**A1**	

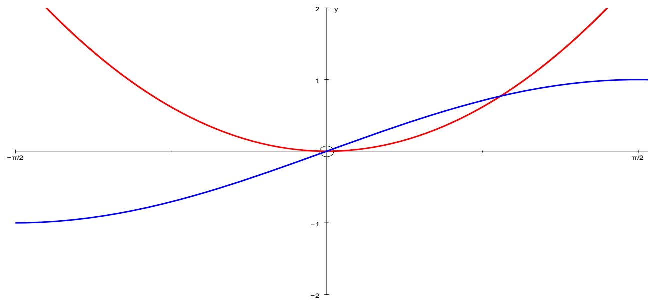
Question	Answer	Partial Marks	Guidance
2	$\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 0$ and $\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = 0$	**M1**	Attempt two dot products equal to 0
	$a - b = -1$ and $-2a + b = -1$	**M1**	Form two simultaneous equations, and attempt to solve
	$a = 2$, $b = 3$	**A1**	Obtain both correct values

Question	Answer	Partial Marks	Guidance
3	$(2-y)^2+y^2=16$ or $x^2+(2-x)^2=16$	**M1**	Substitute to obtain an eqn. in one variable
	Obtain 3 term quadratic $x^2-2x-6=0$	**A1**	Or $y^2-2y-6=0$
	$x=\dfrac{2\pm\sqrt{28}}{2}$	**M1**	Attempt to solve quadratic e.g. complete the square or use the formula If using formula it must be correct
	Obtain $x=1\pm\sqrt{7}$	**A1**	Or equivalent y
	Obtain $(1-\sqrt{7},1+\sqrt{7})$ and $(1+\sqrt{7},1-\sqrt{7})$	**A1**	Must be clearly correct pairings of (x, y) values

Question	Answer	Partial Marks	Guidance
4(a)	$R^2 = 6^2 + (\sqrt{13})^2$	**M1**	Attempt R, using Pythagoras or any other valid method
	$R = 7$	**A1**	Obtain $R = 7$
	$\tan\alpha=\dfrac{\sqrt{13}}{6}$ or $\cos\alpha=\dfrac{6}{7}$ or $\sin\alpha=\dfrac{\sqrt{13}}{7}$	**M1**	Attempt $\tan\alpha=k$, using correct identity for $R\sin(x+\alpha)$ Could use equiv. method if R already found
	$\alpha=31^\circ$	**A1**	Obtain $\alpha=31^\circ$, or better
4(b)	$x+31^\circ = 90^\circ$	**M1**	Identify that $x+\alpha=90^\circ$
	$x = 59^\circ$	**A1FT**	Obtain $x = 59^\circ$ FT as 90° – *their* α
4(c)	–8 is below the minimum value of –7	**B1**	Allow alt. explanations

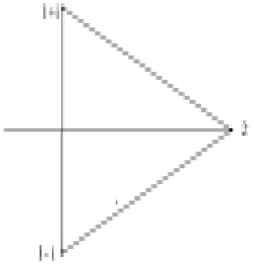
Question	Answer	Partial Marks	Guidance
5(a)	$\int_0^2 g(x) = \pm 4 \quad \int_2^3 g(x) = 2$	M1	Attempt areas of both regions, by triangles or calculus (integrands are –4*x* and 4*x* – 8)
	$\int_0^2 g(x) + \int_2^3 g(x) = -4 + 2 = -2$	A1	Correctly combine integrals to obtain 2
	Alternative Equal area above and below the *x*-axis for the second line segment hence only need $\int_0^1 g(x) = -2$	(M1) (A1)	M1 attempt A1 obtain –2
5(b)	$\mathrm{h}'(x) = \frac{\mathrm{g}'(x)}{\mathrm{g}(x)}$ or by finding h(*x*) = ln(4*x* – 8) directly	M1	Attempt use of chain rule on ln(g(*x*)) or on h(*x*) = ln(4*x* – 8) Allow use of incorrect ln(*ax* + *b*) but derivative must have 1/this
	$\mathrm{h}'(x) = \frac{4}{4x - 8}$	A1	Obtain correct derivative
	$\mathrm{h}'(2.5) = \frac{\mathrm{g}'(2.5)}{\mathrm{g}(2.5)} = \frac{4}{2} = 2$	A1	Substitute *x* = 2.5 to obtain 2
5(c)	Translation of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	B1	Allow equivalent descriptions such as translation of 2 units parallel to the (positive) *x*-axis and 1 unit parallel to the (positive) *y*-axis Do not allow informal language such as 'move, shift, in, on'

Question	Answer	Partial Marks	Guidance
6	$(x+1)^3 = x^3 + 3x^2 + 3x + 1$	**B1**	Correct expansion soi
	$(\frac{1}{x} - 3x)^6 = \frac{1}{x^6} + 6(\frac{1}{x^5})(-3x)^1 + 15(\frac{1}{x^4})(-3x)^2 +$ $20(\frac{1}{x^3})(-3x)^3 + 15(\frac{1}{x^2})(-3x)^4 + 6(\frac{1}{x^1})(-3x)^5 + (-3x)^6$	**M1**	A clear attempt at a binomial expansion or attempt to locate the relevant term: $15(\frac{1}{x^4})(-3x)^2$ unsimplified
	Obtain $\frac{135}{x^2}$	**A1**	Coefficient correct. Ignore any irrelevant terms
	$3x^2 \times \left(-\frac{18}{x^4}\right)$	**M1**	Attempt at product to find other relevant term. If full expansion done then this term must be clearly identified/used for the M1
	$= \frac{-54}{x^2}$	**A1**	Obtain correct coefficient
	coefficient is –54 + 135 = 81	**A1**	Condone $\frac{81}{x^2}$ or 81 x^{-2}

Question	Answer	Partial Marks	Guidance
7(a)		B1	Both graphs correct and shown for at least $-\frac{1}{2}\pi \le x \le \frac{1}{2}\pi$ Graphs must cross in the first quadrant
	Only one intersection point in the positive quadrant hence one positive root	B1	Any equivalent statement Accept an arrow marking the intersection or some other indication. B0 for 'only one point of intersection' unless $x = 0$ clearly discounted
7(b)	$x_{n+1} = x_n - \dfrac{\sin x_n - x_n^2}{\cos x_n - 2x_n}$	M1	Attempt N-R formula, from using $\mathrm{f}(x) = \sin x - x^2$
		A1	Obtain correct N-R formula
	$x_1 = 0.891$ $x_2 = 0.877$ $x_3 = 0.877$	A1	Must be given to at least 3 s.f.
7(c)	f(0.8765) = 0.000252 f(0.8775) = –0.000863	M1	Substitute 0.8775 and 0.8765 in $\sin x - x^2$ Could use any two suitable values either side of 0.877 Could attempt $\sin 0.8775 < 0.8775^2$ and $\sin 0.8765 > 0.8765^2$ **FT** *their* 'α' (NB use of degrees leads to 0.0176)
	$-0.000863 < 0$ and $0.000252 > 0$ sign change hence $0.8765 < \alpha < 0.8775$ (so $\alpha = 0.877$ to 3 s.f.)	A1	Must be correct values, with some conclusion Could be a word statement about crossing the axis or arrows pointing clearly to the different signs to indicate that a root exists between the two values

Question	Answer	Partial Marks	Guidance
8(a)	$A = (3a, 0)$	**B1***	Condone no 'A =', but B0 if given as B
	$B = \left(0, \dfrac{3}{a^2+1}\right)$	**B1***	Condone 'no B = ', but B0 if given as A Allow unsimplified version $\left(0, \dfrac{3a}{a(a^2+1)}\right)$
	Area = $= \dfrac{1}{2} \times 3a \times \dfrac{3}{a^2+1} = \dfrac{9a}{2(a^2+1)}$ **A.G.**	**B1dep***	Attempt area of triangle to clearly obtain the given answer SR B1 only if A and B transposed but correct area then found
8(b)	$\dfrac{\mathrm{d}A}{\mathrm{d}a} = \dfrac{18(a^2+1) - 9a(4a)}{4(a^2+1)^2}$	**M1**	Correct use of quotient rule
		A1	Correct derivative aef inc unsimplified
	$18 = 18a^2$	**M1**	Put derivative equal to 0
	$a = 1$	**A1**	Obtain a = 1 from correct working A0 for a = ± 1 unless –1 later discounted
	area = $\dfrac{9}{4}$	**A1**	
	$\dfrac{\mathrm{d}^2A}{\mathrm{d}a^2} = \dfrac{-36a \times 4(a^2+1)^2 - (18-18a^2)(16a(a^2+1))}{16(a^2+1)^4}$	**M1**	Attempt at second derivative, or test gradient either side of *their a*
	at a = 1, $\dfrac{\mathrm{d}^2A}{\mathrm{d}a^2} = -\dfrac{9}{4} < 0$ hence maximum	**A1**	Obtain appropriate value(s) or clearly demonstrate sign(s) and conclude as maximum

Question	Answer	Partial Marks	Guidance
9(a)	root = 1 + i	**B1**	Correct second root soi
	$(z-(1+\mathrm{i}))(z-(1-\mathrm{i}))$	**M1**	Multiply two factors together
	z^2-2z+2	**A1**	Obtain correct quadratic factor
	$(2z^3-7z^2+10z-6)=(z^2-2z+2)(2z-3)$	**M1**	Attempt $(2z^3-7z^2+10z-6)\div(z^2-2z+2)$
	$z=\dfrac{3}{2}$	**A1**	Correct third root from completely correct working
	Alternative MS for last 4 marks using roots of polynomials:	**(M1)**	attempt sum/pairs/product of cubic
		(A1)	correct (unsimplified) equation
		(M1)	solve to find third root
		(A1)	obtain $z=\dfrac{3}{2}$

Question	Answer	Partial Marks	Guidance
9(b)	$z^* = 1 + \mathrm{i}$	**B1**	Correct z^*
	$zz^* = 2$	**B1**	Correct zz^*
		**B1FT**	All three points correctly plotted and labelled (allow implied labelling from clear scales on both axes)
	$PQ = 2$, $PR = QR = \sqrt{2}$ so ΔPQR is Rt.$\angle^{\mathrm{d}}$ isosceles OR via $m_{PR} = 1$, $m_{QR} = -1$ so $\angle PRQ = 90°$ OR via line of symmetry OR so $\angle PRQ = 2 \times \tan^{-1} 1$ etc.	**M1**	Attempt angle PRQ. Could use symmetry, gradients of lines, cosine rule or note that $OPRQ$ is a square.
		A1	Obtain angle $PRQ = 90°$ **CWO** **SC** B1 only for their correct angle but without clear justification

Question	Answer	Partial Marks	Guidance
10(a)	$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$	**B1**	Use addition identity
	$(2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$	**M1**	Use at least one double angle formula correctly
	$2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$	**M1**	Use Pythagorean identity
	$2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ $4\cos^3\theta - 3\cos\theta$ **A.G.**	**A1**	Expand and obtain given answer – detail needed

Question	Answer	Partial Marks	Guidance
10(b)	$4\cos\theta(\cos\frac{\pi}{3}\cos\theta+\sin\frac{\pi}{3}\sin\theta)(\cos\frac{\pi}{3}\cos\theta-\sin\frac{\pi}{3}\sin\theta)$	**B1**	Use of correct addition identity for both relevant terms
	$4\cos\theta(\frac{1}{2}\cos\theta+\frac{\sqrt{3}}{2}\sin\theta)(\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta)$	**M1**	Use of exact values for $\sin\frac{\pi}{3}$ and $\cos\frac{\pi}{3}$. Condone use in an incorrect expansion, as long as both exact values used
	$4\cos\theta(\frac{1}{4}\cos^2\theta-\frac{3}{4}\sin^2\theta)$	**M1**	Use of difference of two squares or multiplication of the brackets
	$4\cos\theta(\frac{1}{4}\cos^2\theta-\frac{3}{4}(1-\cos^2\theta))$	**M1**	Use Pythagorean identity and attempt to simplify
	$\cos^3\theta-3\cos\theta+3\cos^2\theta$ $4\cos^3\theta-3\cos\theta$ **A.G.**	**A1**	Simplify correctly and conclude appropriately Answer given, so detail needed

Question	Answer	Partial Marks	Guidance
11(a)	$\dfrac{(u-1)^2+(u+1)(u-1)^2+(u+1)^2-(u-1)(u+1)^2}{(u+1)^2(u-1)^2}$	**M1***	Attempt to write as a single fraction with a common denominator of $(u+1)^2(u-1)^2$, which may be implied by following working Could simplify to $\dfrac{u+2}{(u+1)^2}+\dfrac{2-u}{(u-1)^2}$ first, but M1 only awarded when these fractions are combined
		A1	Obtain correct (unsimplified) single fraction
	Numerator = $\left(u^2-2u+1\right)+\left(u^3-u^2-u+1\right)$ $+\left(u^2+2u+1\right)+\left(u^3+u^2-u-1\right)$	**M1dep***	Expand numerator
		A1	Obtain correct (unsimplified) numerator
	$\dfrac{4}{(u+1)^2(u-1)^2}$ hence $k=4$	**A1**	Simplify to obtain $k=4$
11(b)	$2u\dfrac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$	**M1**	Attempt to link dx and du Could rearrange first and differentiate $x=\ln(u^2-1)$
	$\mathrm{d}x=\dfrac{2u}{u^2-1}\,\mathrm{d}u$ or $du=\frac{1}{2}\mathrm{e}^x\left(\mathrm{e}^x+1\right)^{-\frac{1}{2}}\,\mathrm{d}x$	**A1**	Any correct relationship
	$\displaystyle\int\frac{2u}{(u^2-1)u(u^2-1)}\mathrm{d}u \Rightarrow \int\frac{2}{(u^2-1)^2}\mathrm{d}u$	**M1**	Substitute to obtain integral in u
	$\displaystyle\int\frac{2}{[(u+1)(u-1)]^2}\mathrm{d}u=\int\frac{2}{(u+1)^2(u-1)^2}\mathrm{d}u$ **A.G.**	**A1**	Given answer obtained by use of difference of two squares (possibly implicitly)

Question	Answer	Partial Marks	Guidance
11(c)		**M1***	Attempt at integration using multiple of given initial form
	$\frac{1}{2}\left(-\frac{1}{(u+1)}+\ln\lvert u+1\rvert-\frac{1}{(u-1)}-\ln\lvert u-1\rvert\right)$	**A1**	Obtain correct integral – allow A1 if factor of $\frac{1}{2}$ omitted or incorrect. Brackets okay instead of modulus signs in the logs
	$\frac{1}{2}\left[\left(-\frac{1}{4}+\ln 4-\frac{1}{2}-\ln 2\right)-\left(-\frac{1}{3}+\ln 3-1-\ln 1\right)\right]$	**M1dep***	Attempted use of correct limits – correct order and subtraction: must be u limits in F(u) or x limits in F(x). NB ln1 = 0 may not be seen.
	$\frac{1}{2}\left[\left(\ln 2-\frac{3}{4}\right)-\left(\ln 3-\frac{4}{3}\right)\right]$	**M1**	Attempt to simplify to required form, including correct use of laws of logs
	$\frac{7}{24}+\frac{1}{2}\ln\frac{2}{3}$ or $\frac{7}{24}-\frac{1}{2}\ln\frac{3}{2}$	**A1**	Obtain answer in any suitable, correct form