## Cambridge Pre-U

## MATHEMATICS

You must answer on the answer booklet/paper.
You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

## INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do not use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do not use staples, paper clips or glue.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].

1 Two points $A$ and $B$ have coordinates $(1,2)$ and $(7,10)$ respectively. Given that $A B$ is a diameter of a circle, find the equation of the circle.

2 (a) Find the discriminant of $x^{2}+k x+2 k-3$, where $k$ is a constant.
(b) The equation $x^{2}+k x+2 k-3=0$ has two distinct real roots. Find the set of possible values of $k$.

3 Without using a calculator, express each of the following in the form $a+b \sqrt{3}$, where $a$ and $b$ are integers.
(a) $(4+5 \sqrt{3})(2+\sqrt{27})$
(b) $\frac{12}{3+2 \sqrt{3}}$

4


The diagram shows a sector $O A B$ of a circle, centre $O$ and radius $r \mathrm{~cm}$. The angle $A O B$ is 1.4 radians. The shaded segment enclosed by the $\operatorname{arc} A B$ and the chord $A B$ has an area of $12 \mathrm{~cm}^{2}$.
(a) Find the value of $r$, correct to 4 significant figures.
(b) Find the perimeter of the shaded segment, correct to 3 significant figures.

5 A curve has equation $y=\mathrm{e}^{2 x}+1$. The region $R$ is bounded by the curve, the $x$-axis, the $y$-axis and the line $x=2$. Find the exact volume when $R$ is rotated $360^{\circ}$ around the $x$-axis.

6 A sequence $u_{1}, u_{2}, u_{3}, \ldots$ is defined by $u_{1}=4$ and $u_{n+1}=u_{n}+3$.
Another sequence $v_{1}, v_{2}, v_{3}, \ldots$ is defined by $v_{1}=1200$ and $v_{n+1}=0.8 v_{n}$.
(a) Find $u_{20}-v_{20}$, giving your answer correct to 3 significant figures.
(b) Use an algebraic method to find the smallest value of $N$ such that $\sum_{n=1}^{N} u_{n}>\sum_{n=1}^{\infty} v_{n}$.

7 It is given that $\theta$ is the acute angle such that $\sin \theta=\frac{1}{4}$.
(a) Show that $\cos \theta=\frac{\sqrt{15}}{4}$.
(b) Hence, using an appropriate formula in each case, find the exact values of
(i) $\cos \left(\theta-30^{\circ}\right)$,
(ii) $\operatorname{cosec} 2 \theta$.

8 (a) Find the quotient when $3 x^{4}+8 x^{3}-24 x^{2}+22 x+9$ is divided by $x^{2}+4 x-3$, and show that the remainder is $6 x+12$.
(b) Hence find the exact value of $\int_{1}^{3} \frac{3 x^{4}+8 x^{3}-24 x^{2}+22 x+9}{x^{2}+4 x-3} \mathrm{~d} x$. Give your answer in the form $a+b \ln c$, where $a, b$ and $c$ are integers.

9 Two straight lines have equations

$$
\mathbf{r}=\mathbf{i}+4 \mathbf{j}+6 \mathbf{k}+\lambda(2 \mathbf{i}+a \mathbf{j}+\mathbf{k}) \quad \text { and } \quad \mathbf{r}=4 \mathbf{i}+4 \mathbf{j}+9 \mathbf{k}+\mu(\mathbf{i}+b \mathbf{j}+2 \mathbf{k})
$$

where $a$ and $b$ are constants.
(a) Given that the two lines intersect, show that $a+b=0$.
(b) Given also that the angle between the two lines is $60^{\circ}$, find the possible values of $a$ and $b$.

10 A population, $P$, of a certain species at time $t$ is such that the rate of increase of $P$ at any particular time is proportional to $(3 P+50)^{\frac{1}{3}}$. When $t=0, P=25$ and when $t=13, P=154$. Write down a differential equation for this situation and solve it to find $P$ in terms of $t$.

11 A curve has parametric equations $x=\frac{2}{t}-1, y=\ln \left(3 t-t^{2}\right)$, for $0<t<3$.
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of $t$, and hence find the exact coordinates of the stationary point on the curve.
(b) Find $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ in terms of $t$, and hence determine the nature of the stationary point.

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