

Cambridge Pre-U

MATHEMATICS

Paper 2 Pure Mathematics 2

9794/02

October/November 2020

2 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper Graph paper List of formulae (MF20)

INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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1 Find the sum of all the integers between 1 and 200 which are divisible by 7. [3]

2 Given that
$$\begin{pmatrix} a \\ b \\ 1 \end{pmatrix}$$
 is perpendicular to both $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$, find the values of *a* and *b*. [3]

3 The diagram shows the graphs of $x^2 + y^2 = 16$ and y = 2 - x.



Find, in terms of $\sqrt{7}$, the exact coordinates of the points of intersection of the two graphs. [5]

- 4 (a) Express $6 \sin x + \sqrt{13} \cos x$ in the form $R \sin(x + \alpha)$, where R > 0 and $0^{\circ} < \alpha < 90^{\circ}$. [4]
 - (b) Find the smallest positive value of x for which the maximum value of $6 \sin x + \sqrt{13} \cos x$ occurs.

[2]

(c) Explain why the equation $6\sin x + \sqrt{13}\cos x = -8$ has no solution. [1]

5 The diagram shows the graph of y = g(x), $0 \le x \le 3$, which consists of two straight line segments. One line segment joins the points (0, 0) and (1, -4), and the other line segment joins the points (1, -4) and (3, 4).



(a) Find
$$\int_{0}^{3} g(x) dx$$
. [2]

- (b) For $2 < x \le 3$, let $h(x) = \ln(g(x))$. Determine h'(2.5). [3]
- (c) Describe fully the transformation which, when applied to the graph of y = g(x), will produce the graph of y = g(x 2) + 1. [1]

6 Find the coefficient of
$$\frac{1}{x^2}$$
 in the expansion of $(x+1)^3 \left(\frac{1}{x} - 3x\right)^6$. [6]

- 7 (a) By sketching the graphs of $y = x^2$ and $y = \sin x$ on the same diagram, show that the equation $\sin x = x^2$ has exactly one positive root α in the interval $-\frac{1}{2}\pi \le x \le \frac{1}{2}\pi$. [2]
 - (b) Use the Newton-Raphson iteration method with $f(x) = \sin x x^2$ and $x_0 = 1$ to find the values of x_1, x_2 and x_3 , correct to 3 significant figures. [3]
 - (c) Use a change of sign method to verify that $x_3 = \alpha$, correct to 3 significant figures. [2]
- 8 A line has equation $a(a^2 + 1)y = 3a x$, where *a* is positive. This line intersects the *x*-axis at *A* and the *y*-axis at *B*. The origin is *O*.
 - (a) Show that the area of triangle *AOB* is $\frac{9a}{2(a^2+1)}$. [3]
 - (b) Find the maximum possible area of triangle *AOB* as *a* varies. [7]

- 9 (a) Given that 1 i is a root of the equation $2z^3 7z^2 + 10z 6 = 0$, find the other two roots. [5]
 - (b) Given that z = 1 i, write down z^* and find zz^* .

Plot points P, Q and R representing the numbers z, z^* and zz^* respectively on an Argand diagram. Determine the angle PRQ. [5]

- 10 (a) Prove that $\cos 3\theta = 4\cos^3 \theta 3\cos \theta$. [4]
 - (**b**) Prove also that $4\cos\theta\cos(\frac{1}{3}\pi \theta)\cos(\frac{1}{3}\pi + \theta) = 4\cos^3\theta 3\cos\theta.$ [5]
- 11 (a) Show that $\frac{1}{(u+1)^2} + \frac{1}{(u+1)} + \frac{1}{(u-1)^2} \frac{1}{(u-1)} = \frac{k}{(u+1)^2(u-1)^2}$, where k is a constant to be determined. [5]
 - (b) Using the substitution $u^2 = e^x + 1$, show that

$$\int \frac{1}{e^x \sqrt{e^x + 1}} \, \mathrm{d}x = \int \frac{2}{(u+1)^2 (u-1)^2} \, \mathrm{d}u.$$
 [4]

(c) Find the exact value of $\int_{\ln 3}^{\ln 8} \frac{1}{e^x \sqrt{e^x + 1}} dx$, giving your answer in the form $a + b \ln c$, where a, b and c are rational. [5]

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