Cambridge Assessment International Education

Cambridge Pre-U

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

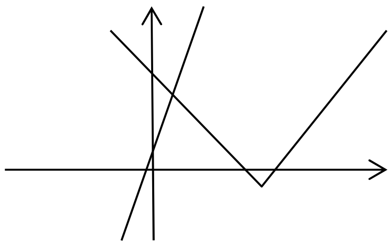
Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1	$\tan 3\theta = \frac{2}{3}$	**B1**	Obtain correct equation
	$3\theta = 33.7°, 213.7°, 393.7°$	**M1**	Attempt to solve $\tan 3\theta = k$, using correct order of operations
	$\theta = 11.2°$	**A1**	Obtain one correct angle
	$\theta = 71.2°, 131.2°$	**A1**	Obtain two further correct angles, and no others in range

Question	Answer	Marks	Guidance
2(a)	$2(x + 2)^2 – 8$	**B1**	Obtain $p = 2$
		B1	Obtain $q = 2$
		B1	Obtain $r = –8$
2(b)	$(x + 2)^2 – 4 \geqslant – 4$	**M1**	Attempt to identify minimum point of $x^2 + 4x$ (completed square form, differentiation etc)
	$k < –4$	**A1**	Obtain $k < –4$

Question	Answer	Marks	Guidance
3(a)	$x^2 + (x + 3)^2 – 7x = 12$	**M1**	Eliminate one variable
	$2x^2 – x – 3 = 0$	**A1**	Obtain correct three term quadratic
	$(2x – 3)(x + 1) = 0$	**M1**	Attempt to solve quadratic
	$x = 1.5, x = –1$	**A1**	Obtain correct x-coordinates
	$y = 4.5, y = 2$	**A1**	Obtain correct y-coordinates

Question	Answer	Marks	Guidance
3(b)	centre is (3.5, 0)	**B1**	Correct centre of circle soi
	$0 = 3.5 + c$	**M1**	Substitute their centre into the equation of the line, or use informal method as $m = 1$
	$c = –3.5$	**A1**	Obtain $c = –3.5$

Question	Answer	Marks	Guidance
4(a)		**M1**	Sketch V-shaped graph with vertex on positive x-axis
		A1	(5, 0) and (0, 5) soi
		B1	Linear graph drawn with (–0.5, 0) and (0, 1) soi For full marks, linear graph must have steeper gradient than positive part of modulus graph,
4(b)	$2x + 1 = 5 – x$	**M1**	Attempt to solve equation in which $2x$ and x have opposite signs, or square both sides of equation and attempt to solve
	$x = \frac{4}{3}$	**A1**	Obtain correct point of intersection BOD if $x = –6$ also given; condone any sign
	$x < \frac{4}{3}$	**A1**	Correct inequality A0 if $x > –6$ also given

Question	Answer	Marks	Guidance
5	$\frac{dy}{dx} = 2x\ln x + x^2\frac{1}{x}$	**M1**	Attempt differentiation using the product rule
	$\frac{dy}{dx} = 2x\ln x + x$	**A1**	Obtain correct derivative
	$x(2\ln x + 1) = 0$	**M1**	Equate to 0 and attempt to solve
	$x = 0$ gives no solution as ln0 is undefined	**B1**	Justify why $x = 0$ is not valid If B0, then other 5 marks are still available
	$\ln x = -\frac{1}{2}$, so $x = e^{-\frac{1}{2}}$	**A1**	Obtain correct x-coordinate; accept any exact equiv No need to explicitly state that this is the only root
	$y = -\frac{1}{2}e^{-1}$	**A1**	Obtain correct y-coordinate; accept any exact simplified equiv

Question	Answer	Marks	Guidance
6	$xy^2 = 8$	**B1**	Correct equation not involving logs
	$\log_2 x^2 + \log_2 y^3 = 4$ $\log_2 x^2 y^3 = 4$	**M1**	Correct use of log laws
	$x^2 y^3 = 16$	**A1**	Correct equation not involving logs
	$\left(\frac{8}{y^2}\right)^2 y^3 = 16$	**M1**	Eliminate one variable and attempt to solve
	$y = 4$	**A1**	Obtain one correct value
	$x = \frac{1}{2}$	**A1**	Obtain other correct value **ALT MS** M1 – correct use of log laws to split first equation into 2 terms (allow $\log_2 y^2$) A1 – obtain $\log_2 x + 2\log_2 y = 3$ M1 – attempt to solve simultaneously A1 – obtain $\log_2 x = -1$ or $\log_2 y = 2$ A1, A1 – as above

Question	Answer	Marks	Guidance
7	$\frac{9}{4}=\frac{3k}{2}$	**M1**	Attempt to find k
	$k=\frac{3}{2}$	**A1**	Obtain $k=\frac{3}{2}$
	$\int(2y)\,dy=\int\left(3x^2-6x\right)dx$	**M1**	Separate variables and attempt integration
	$y^2=x^3-3x^2+c$	**A1FT**	Obtain correct integral, following their k
	$4=27-27+c \Rightarrow c=4$	**M1**	Attempt to find c using (3, 2)
	$y^2=x^3-3x^2+4$	**A1**	Obtain correct equation aef

Question	Answer	Marks	Guidance
8(a)	$\frac{0+6}{3}=2$ and $2-0=2$	**M1**	Use $y=0$ and $z=0$
	consistent, hence crosses x-axis	**A1**	Justify intersection with x-axis
	(9, 0, 0)	**B1**	State correct coordinates (can follow M0)
8(b)	$2\mathbf{i}+3\mathbf{j}-\mathbf{k}$	**B1**	Identify correct direction vector for l_1
	$\lvert 2\mathbf{i}+3\mathbf{j}-\mathbf{k}\rvert=\sqrt{14}$ $\lvert 7\mathbf{i}-3\mathbf{j}+2\mathbf{k}\rvert=\sqrt{62}$	**B1FT**	Correct (unsimplified) magnitude for either of their direction vectors
	$14-9-2=\sqrt{14}\sqrt{62}\cos\theta$	**M1**	Attempt use of correct formula
	$\theta=84.2^\circ$	**A1**	Obtain 84.2° or 1.47 radians

Question	Answer	Marks	Guidance
9(a)	other root is $3 + 2\text{i}$	**B1**	Correct second root soi
	$(z - (3 - 2\text{i}))(z - (3 + 2\text{i})) = z^2 - (3 - 2\text{i})z - (3 + 2\text{i})z + (3 - 2\text{i})(3 + 2\text{i})$ or sum of roots = 6, product of roots = 13	**M1**	Attempt method to find both coefficients
	$a = -6$	**A1**	Obtain $a = -6$ (could be embedded in equation)
	$b = 13$	**A1**	Obtain $b = 13$ (could be embedded in equation) **ALT MS** B1 Substitute to obtain $(5 - 12\text{i}) + (3 - 2\text{i})a + b = 0$ M1 Consider Re and Im parts to attempt a and b A1 A1 as per main MS
9(b)	$c - \text{i} = 6 + 3d\text{i} - 4\text{i} - 2d\text{i}^2$	**M1**	Remove fraction and expand brackets, or multiply numerator and denominator by complex conjugate
	$c = 6 + 2d \quad -1 = 3d - 4$	**M1**	Equate Re and Im parts
	$-1 = 3d - 4 \Rightarrow d = 1$	**A1**	Obtain $d = 1$
	$c = 6 + 2d \Rightarrow c = 8$	**A1**	Obtain $c = 8$
9(c)	$p = 7$	**B1**	Obtain $p = 7$ (could be implied by answer $7 + q\text{i}$)
	$q = -2$	**B1**	Obtain $q = -2$ (could be implied by answer $p - 2\text{i}$)

Question	Answer	Marks	Guidance
10(a)	$3\times\left(\sqrt{3}\tan 2\theta\right)^2 = 27$ $\left(\sqrt{3}\tan 2\theta\right)^2 = 9$	**M1**	Attempt to use $u_3 = ar^2$
	$\tan^2 2\theta = 3$ $\tan 2\theta = \pm\sqrt{3}$ $2\theta = \pm\frac{1}{3}\pi$	**M1**	Attempt to solve equation (condone no ±)
	$\theta = \frac{1}{6}\pi$	**A1**	Obtain one correct root
	$\theta = -\frac{1}{6}\pi$	**A1**	Obtain both correct roots, and no others
10(b)	$-1 < \sqrt{3}\tan 2\theta < 1$ or $\left\lvert\sqrt{3}\tan 2\theta\right\rvert$	**B1**	Identify that $-1 < r < 1$
	$-\frac{1}{\sqrt{3}} < \tan 2\theta < \frac{1}{\sqrt{3}}$ $-\frac{1}{6}\pi < 2\theta < \frac{1}{6}\pi$	**M1**	Link $\sqrt{3}\tan 2\theta$ to 1 and/or –1 (any inking sign), and attempt to solve
	$-\frac{1}{12}\pi < \theta < \frac{1}{12}\pi$	**A1**	Obtain correct inequality

Question	Answer	Marks	Guidance
10(c)	$\dfrac{3}{1-r}=\dfrac{3\left(2+\sqrt{3}\right)}{2}$	**B1**	Correct equation, with either r or the actual ratio either in terms of θ, or with $\frac{1}{24}\pi$ substituted
	$1-r=\dfrac{2}{2+\sqrt{3}}$ $r=1-\dfrac{2}{2+\sqrt{3}}$ $r=\dfrac{\sqrt{3}}{2+\sqrt{3}}$	**M1**	Attempt to make r the subject of the equation – award this mark when $r =$ … is obtained
	$r=\dfrac{\sqrt{3}\left(2-\sqrt{3}\right)}{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}$	**M1**	Rationalise their two term denominator – may come before previous M1 Must be seen explicitly – if not seen explicitly then B1 M1 M0 A1 A1 can still be awarded
	$r=\dfrac{\sqrt{3}\left(2-\sqrt{3}\right)}{4-3}=\sqrt{3}\left(2-\sqrt{3}\right)$	**A1**	Obtain correct equation with r as the subject
	$\sqrt{3}\tan\left(2\times\dfrac{1}{24}\pi\right)=\sqrt{3}\left(2-\sqrt{3}\right)$ $\tan\left(\dfrac{1}{12}\pi\right)=2-\sqrt{3}$ **A.G.**	**A1**	Obtain given answer

Question	Answer	Marks	Guidance
11	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2-4x+1)-(x-2)(2x-4)}{(x^2-4x+1)^2}$	M1	Attempt differentiation using quotient rule
		A1	Correct derivative, allow unsimplified
	when $x=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-1$ $y-\frac{1}{2}=-1(x-1)$ $y=-x+\frac{3}{2}$	M1	Attempt equation of tangent – can follow M0, as long as using attempt at derivative with $x = 1$
	tangent crosses x-axis at $\left(\frac{3}{2},0\right)$	A1	Correct x intersection
	$\int_1^2 \frac{x-2}{x^2-4x+1}\,\mathrm{d}x=\left[\frac{1}{2}\ln\left\|x^2-4x+1\right\|\right]_1^2$	M1	Attempt integration
		A1	Correct integral
	$\frac{1}{2}\ln\|-3\|-\frac{1}{2}\ln\|-2\|$	M1	Use limits of 1 and 2 – correct order and subtraction
	$=\frac{1}{2}\ln 3-\frac{1}{2}\ln 2=\frac{1}{2}\ln\frac{3}{2}$	A1	Correct area
	area of triangle = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$	M1	Attempt complete method to find shaded area
	shaded area = $\frac{1}{2}\ln\frac{3}{2}-\frac{1}{8}$	A1	Obtain correct area – any simplified exact equiv