Cambridge Pre-U

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2023**

2 hours

* 1 3 4 3 7 2 5 8 1 7 *

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS
- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION
- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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1 Solve the equation $3\sin 3\theta = 2\cos 3\theta$ for $0° < \theta < 180°$. [4]

2 **(a)** Express $2x^2 + 8x$ in the form $p(x+q)^2 + r$, where p, q and r are constants. [3]

(b) Given that $x^2 + 4x > k$ for all values of x, state the set of values of k. [2]

3 A circle has equation $x^2 + y^2 - 7x = 12$.

(a) Find the coordinates of the points of intersection of the line $y = x + 3$ and this circle. [5]

(b) The equation of a diameter of this circle is $y = x + c$. Find the value of the constant c. [3]

4 **(a)** On the same axes sketch the graphs of $y = 2x + 1$ and $y = |x - 5|$. [3]

(b) Solve the inequality $2x + 1 < |x - 5|$. [3]

5 Show that the curve $y = x^2 \ln x$ has only one stationary point, and find its exact coordinates. [6]

6 Solve the simultaneous equations $\log_2(xy^2) = 3$ and $2\log_2 x + 3\log_2 y = 4$. [6]

7 The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{kx(x-2)}{y}$, where k is a constant. The curve passes through the point $(3, 2)$, and the gradient of the curve at this point is $\frac{9}{4}$. Find the equation of the curve, giving your answer in the form $\mathrm{f}(y) = \mathrm{g}(x)$. [6]

8 A line has equation $\frac{x-5}{2} = \frac{y+6}{3} = 2 - z$.

(a) Show that this line intersects the x-axis, and state the coordinates of the point of intersection. [3]

A second line has equation $\mathbf{r} = (3 + 7\mu)\mathbf{i} + (2 - 3\mu)\mathbf{j} + (-4 + 2\mu)\mathbf{k}$.

(b) Find the acute angle between the two lines. [4]

9 **(a)** Find the values of the real constants a and b, such that $3 - 2\mathrm{i}$ is a root of the equation $z^2 + az + b = 0$. [4]

(b) Find the values of the real constants c and d, such that $\frac{c - \mathrm{i}}{2 + d\mathrm{i}} = 3 - 2\mathrm{i}$. [4]

(c) State the values of the real constants p and q, such that the locus $|z - (3 - 2\mathrm{i})| = |z - (p + q\mathrm{i})|$ has equation $\mathrm{Re}(z) = 5$. [2]

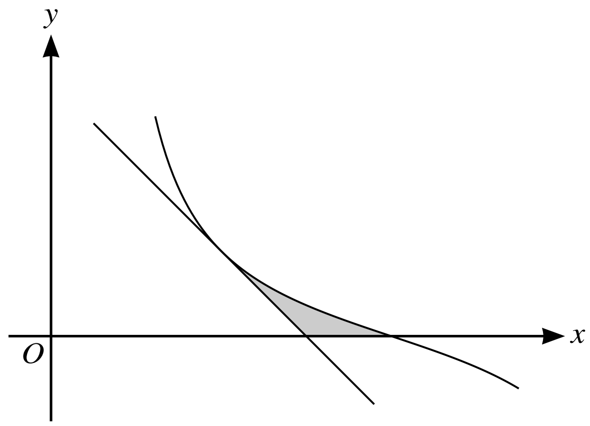
10 In a geometric progression the first term is 3 and the common ratio is $\sqrt{3}\tan 2\theta$, where $-\frac{1}{4}\pi < \theta < \frac{1}{4}\pi$.

(a) Given that the third term is 27, find the possible values of θ. [4]

(b) Given instead that the geometric progression converges, find the set of possible values of θ. [3]

(c) It is given that when $\theta = \frac{1}{24}\pi$ the sum to infinity of the geometric progression is $\frac{3}{2}(2 + \sqrt{3})$. Hence show that $\tan\left(\frac{1}{12}\pi\right) = 2 - \sqrt{3}$. [5]

11



The diagram shows part of the curve $y = \dfrac{x-2}{x^2 - 4x + 1}$, and the tangent to the curve at the point $\left(1, \frac{1}{2}\right)$. Find the exact area of the shaded region enclosed by this part of the curve, the tangent and the x-axis. [10]

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