## Cambridge Pre-U

## MATHEMATICS

You must answer on the answer booklet/paper.
You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

## INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do not use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do not use staples, paper clips or glue.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 4 pages. Any blank pages are indicated.

1 Solve the equation $3 \sin 3 \theta=2 \cos 3 \theta$ for $0^{\circ}<\theta<180^{\circ}$.

2 (a) Express $2 x^{2}+8 x$ in the form $p(x+q)^{2}+r$, where $p, q$ and $r$ are constants.
(b) Given that $x^{2}+4 x>k$ for all values of $x$, state the set of values of $k$.

3 A circle has equation $x^{2}+y^{2}-7 x=12$.
(a) Find the coordinates of the points of intersection of the line $y=x+3$ and this circle.
(b) The equation of a diameter of this circle is $y=x+c$. Find the value of the constant $c$.

4 (a) On the same axes sketch the graphs of $y=2 x+1$ and $y=|x-5|$.
(b) Solve the inequality $2 x+1<|x-5|$.

5 Show that the curve $y=x^{2} \ln x$ has only one stationary point, and find its exact coordinates.

6 Solve the simultaneous equations $\log _{2}\left(x y^{2}\right)=3$ and $2 \log _{2} x+3 \log _{2} y=4$.

7 The gradient of a curve is given by $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{k x(x-2)}{y}$, where $k$ is a constant. The curve passes through the point $(3,2)$, and the gradient of the curve at this point is $\frac{9}{4}$. Find the equation of the curve, giving your answer in the form $\mathrm{f}(y)=\mathrm{g}(x)$.

8 A line has equation $\frac{x-5}{2}=\frac{y+6}{3}=2-z$.
(a) Show that this line intersects the $x$-axis, and state the coordinates of the point of intersection.

A second line has equation $\mathbf{r}=(3+7 \mu) \mathbf{i}+(2-3 \mu) \mathbf{j}+(-4+2 \mu) \mathbf{k}$.
(b) Find the acute angle between the two lines.

9 (a) Find the values of the real constants $a$ and $b$, such that $3-2 \mathrm{i}$ is a root of the equation $z^{2}+a z+b=0$.
(b) Find the values of the real constants $c$ and $d$, such that $\frac{c-\mathrm{i}}{2+d \mathrm{i}}=3-2 \mathrm{i}$.
(c) State the values of the real constants $p$ and $q$, such that the locus $|z-(3-2 \mathrm{i})|=|z-(p+q \mathrm{i})|$ has equation $\operatorname{Re}(z)=5$.

10 In a geometric progression the first term is 3 and the common ratio is $\sqrt{3} \tan 2 \theta$, where $-\frac{1}{4} \pi<\theta<\frac{1}{4} \pi$.
(a) Given that the third term is 27 , find the possible values of $\theta$.
(b) Given instead that the geometric progression converges, find the set of possible values of $\theta$. [3]
(c) It is given that when $\theta=\frac{1}{24} \pi$ the sum to infinity of the geometric progression is $\frac{3}{2}(2+\sqrt{3})$. Hence show that $\tan \left(\frac{1}{12} \pi\right)=2-\sqrt{3}$.

11


The diagram shows part of the curve $y=\frac{x-2}{x^{2}-4 x+1}$, and the tangent to the curve at the point $\left(1, \frac{1}{2}\right)$. Find the exact area of the shaded region enclosed by this part of the curve, the tangent and the $x$-axis.

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