

Cambridge Pre-U

MATHEMATICS

Paper 2 Pure Mathematics 2

9794/02

May/June 2023

2 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper Graph paper List of formulae (MF20)

INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document has **4** pages. Any blank pages are indicated.

- 1 (a) Show that x + 1 is a factor of $4x^3 + 8x^2 + x 3$. [2]
 - (b) Hence solve the equation $4x^3 + 8x^2 + x 3 = 0.$ [4]
- 2 In an arithmetic progression, the first term is 32, the last term is 14 and the sum of the terms is 299. Find the number of terms and the common difference. [4]
- **3** Functions f and g are defined for all real values of x by $f(x) = x^2 + 2$ and g(x) = 3x + 5.
 - (a) State the range of f and the range of g. [2]
 - (b) Find the value of gf(4). [2]
 - (c) State, with a reason, which of f and g does not have an inverse. [1]
- 4 Solve the equation $\cos 2\theta = 2\cos \theta$ for $0^\circ < \theta < 360^\circ$. [5]
- 5 A curve is given parametrically by $x = 3t \ln(3t)$, $y = t^3 \ln(t^3)$, where t > 0. Find the exact value of *t* at the point on the curve at which the gradient is 3. [5]
- 6 (a) Find $\int x e^{2x} dx$. [3]

(b) Determine, in exact form,
$$\int_{\frac{1}{12}\pi}^{\frac{1}{6}\pi} \cos^2(3x) \, dx.$$
 [5]

- 7 (a) Expand $\frac{1}{\sqrt{1+2x}}$ as a series of ascending powers of *x*, up to and including the term in x^3 . Express each coefficient in its simplest form. [4]
 - (b) Hence determine the coefficient of x^2 in the expansion of $\frac{(2-3x)^2}{\sqrt{1+2x}}$. [3]

8 (a) Express
$$\frac{2x^2 + 1}{x(x-1)^2}$$
 as the sum of three partial fractions. [5]

(b) Hence show that
$$\int_{2}^{4} \frac{2x^2 + 1}{x(x-1)^2} dx = p + \ln q$$
, where p and q are integers to be found. [5]

- 9 (a) Give full details of a sequence of transformations which maps the graph of $y = e^x$ onto the graph of $y = e^{2-x}$. [2]
 - (b) Sketch, on the same diagram, the graphs of $y = e^{2-x}$ and $y = \ln(x+1)$, and hence show that the equation $e^{2-x} = \ln(x+1)$ has only one root. [3]
 - (c) This root may be found using the iterative formula $x_{n+1} = 2 \ln(\ln(x_n + 1))$. Using a starting value of 1.5, and showing the result of each iteration, find the value of this root correct to 3 significant figures. [3]
- 10 The equation of a curve C is $x^2 + 2xy + 3y^2 = 18$.
 - (a) Show that the equation of the normal to C at the point (5, -1) can be written as x = 2y + 7. [6]
 - (b) Find the exact coordinates of the point where this normal meets *C* again. [4]

11 The fixed line *L* has vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$.

For each value of the parameter *t*, the line M(t) is given by the equation $\mathbf{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ t \end{pmatrix}$.

- (a) Determine the value of *t* for which
 - (i) L and M(t) are perpendicular;
 - (ii) L and M(t) intersect.

(**b**) The point P(30, 3, 12) is on L and the line M(-1) has equation $\mathbf{r} = \begin{pmatrix} 7+3\mu \\ 1+2\mu \\ 2-\mu \end{pmatrix}$.

The point Q lies on M(-1).

- (i) Show that $(PQ)^2 = 14\mu^2 126\mu + 633.$ [2]
- (ii) Given now that Q is the point on M(-1) which is closest to P, find the coordinates of Q.

[2]

[5]

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