## Cambridge Pre-U

## MATHEMATICS

Paper 3 Applications of Mathematics
May/June 2023
2 hours
You must answer on the answer booklet/paper.
You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

## INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs
- Write your name, centre number and candidate number on all the work you hand in.
- Do not use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity is needed, use $10 \mathrm{~m} \mathrm{~s}^{-2}$.
- At the end of the examination, fasten all your work together. Do not use staples, paper clips or glue.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 8 pages. Any blank pages are indicated.

## Section A: Probability (40 marks)

## You are advised to spend no more than 1 hour on this section.

1 The score, $X$, obtained on a throw of a biased five-sided die is given by the probability distribution

$$
\mathrm{P}(X=r)=k r^{2} \text { for } r=1,2,3,4,5
$$

(a) Show that $k=\frac{1}{55}$.
(b) The die is thrown once. Find the probability that the score obtained is greater than $\mathrm{E}(X)$.

2 A woman makes phone calls at random to people in the hope of recruiting members to a new gym. For each new member recruited, she earns $£ 12$. The probability that the person she phones will become a member is 0.16 . People decide whether or not to become members independently of one another.
(a) Find the probability that she will recruit 2 or more new members from 10 phone calls.
(b) Calculate how much she would earn on average from 20 phone calls.
(c) Calculate the number of phone calls she must make to be at least $95 \%$ certain of recruiting at least 1 new member.

3 (a) A random variable $X$ is normally distributed such that the mean is 4 times the standard deviation. Given that $\mathrm{P}(X>3.6)=0.99$, find the standard deviation of $X$.
(b) A random variable $Y$ is normally distributed. Find the probability that $Y$ is within 1.5 standard deviations of its mean.

4 A student carries out an experiment to determine the solubility of a salt at different temperatures. His data is shown in the table below and the accompanying scatter diagram.

| Temperature $t^{\circ} \mathrm{C}$ | 13 | 22 | 27 | 24 | 29 | 33 | 35 | 45 | 47 | 56 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Solubility $s$ grams/litre | 371 | 376 | 381 | 384 | 387 | 392 | 397 | 401 | 406 | 410 |



The data is summarized as follows.

$$
n=10, \quad \Sigma s=3905, \quad \Sigma s^{2}=1526413, \quad \Sigma t=331, \quad \Sigma t^{2}=12483, \quad \Sigma s t=130739
$$

(a) Calculate the equation of the regression line of $s$ on $t$.
(b) Calculate the residual at $t=35$, and comment on its sign.

Phillip has seven tiles.

(a) Phillip arranges the tiles at random in a line. Find the probability that he arranges them in the correct order to spell his name.
(b) Phillip chooses four of the seven tiles at random. Find the probability that he chooses P, H, I and L in any order.

6 Two events $A$ and $B$ are such that $\mathrm{P}(A \mid B)=2 \times \mathrm{P}(A)$, where $\mathrm{P}(A) \neq 0$.
(a) Explain whether or not $A$ and $B$ are independent events.

It is also given that $\mathrm{P}(A \cap B)=\frac{1}{3} \mathrm{P}(A)$ and that $\mathrm{P}\left(A^{\prime} \cap B^{\prime}\right)=\frac{4}{5}$.
(b) Find $\mathrm{P}(A)$.

## Section B: Mechanics (40 marks)

## You are advised to spend no more than 1 hour on this section.

7 A car is towing a caravan along a straight horizontal road. The car and the caravan are connected by a light horizontal coupling. The car has mass 1400 kg and the resistance to its motion is 200 N . The caravan has mass 600 kg and the resistance to its motion is 300 N . The car and the caravan are accelerating at $1.5 \mathrm{~m} \mathrm{~s}^{-2}$.
(a) Calculate the driving force of the car.
(b) Calculate the tension in the coupling.

8 A particle of mass 0.3 kg is moving down a line of greatest slope of a rough plane inclined at $35^{\circ}$ to the horizontal. The particle has acceleration $2.3 \mathrm{~m} \mathrm{~s}^{-2}$.

Find the coefficient of friction between the particle and the plane.

9 A particle $P$ is projected vertically upwards from horizontal ground with speed $20 \mathrm{~m} \mathrm{~s}^{-1}$. At the same instant, a particle $Q$ is projected vertically upwards with speed $8 \mathrm{~m} \mathrm{~s}^{-1}$ from a point 15 m above the ground.
(a) Find the velocities of $P$ and $Q$ at the instant when they are at the same height.
(b) Find the length of time for which $P$ is higher than $Q$ and $P$ is rising.

10 (a) A particle is projected with speed $U \mathrm{~m} \mathrm{~s}^{-1}$ at angle $\theta$ above the horizontal from a point $O$ on horizontal ground. At time $t$ seconds after projection, the horizontal and vertically upwards displacements of the particle from $O$ are $x \mathrm{~m}$ and $y \mathrm{~m}$ respectively.

Express $x$ and $y$ in terms of $t$, and hence show that the equation of trajectory of the particle is

$$
\begin{equation*}
y=x \tan \theta-\frac{5 x^{2} \sec ^{2} \theta}{U^{2}} \tag{4}
\end{equation*}
$$

(b) A woman is standing 4 m away from a wall, which is 5 m high. She throws a small ball with speed $10 \mathrm{~m} \mathrm{~s}^{-1}$ at angle $\theta$ above the horizontal from a point 2 m above the ground.

Given that the ball clears the wall, find the range of possible values for $\tan \theta$.

11 A particle is travelling in a straight line. Its velocity $v \mathrm{~ms}^{-1}$ at time $t$ seconds is given by

$$
v=-2+4.5 t-t^{2}
$$

(a) State the initial velocity of the particle.
(b) Find the values of $t$ for which the particle is at rest.
(c) Calculate the total distance travelled by the particle in the first 5 seconds.

12 Two particles $A$ and $B$, of masses $m \mathrm{~kg}$ and $5 m \mathrm{~kg}$ respectively, are at rest on a smooth horizontal surface. $A$ is projected with velocity $u \mathrm{~m} \mathrm{~s}^{-1}$ directly towards $B$. The particles collide and the coefficient of restitution in this collision is $e$.
(a) Show that, after the collision, the velocity of $A$ is $\frac{1}{6}(1-5 e) u \mathrm{~m} \mathrm{~s}^{-1}$, and find an expression for the velocity of $B$ in terms of $e$.
(b) Find the set of values of $e$ for which the speed of $A$ is greater than the speed of $B$ after the collision.

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