Numerical Methods

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Numerical Methods
Booklet	Question Paper

Time Allowed: 35   minutes

Score: /29

Percentage: /100

Grade Boundaries:

1   Given that the equation $x^3 + 2x - 7 = 0$ has a root between $x = 1$ and $x = 2$, use the Newton-Raphson formula with $x_0 = 1$ to find this root correct to 3 decimal places. [4]

2   Taking $x = 2$ as a f rst approximation, use the Newton-Raphson process to f nd a root of the equation $\dfrac{1}{x^2} - 0.119 - 0.018x = 0$. Give your answer correct to 3 signif cant f gures. [4]

3   **(i)** Let $f(x) = x^3 - x - 1$. Use a sign change method to show that the equation $x^3 - x - 1 = 0$ has a root between $x = 1$ and $x = 2$. [2]

   (ii) By taking $x = 1$ as a first approximation to this root, use the Newton-Raphson formula to find this root correct to 3 decimal places. [4]

4   **(i)** Show that the equation $x^3 - 6x + 2 = 0$ has a root between $x = 0$ and $x = 1$. [2]

   (ii) Use the iterative formula $x_{n+1} = \dfrac{2 + x_n^3}{6}$ with $x_0 = 0.5$ to fin this root correct to 4 decimal places, showing the result of each iteration. [3]

5   **(i)** Sketch, on a single diagram, the graphs of $y = e^{\frac{1}{5}x}$ and $y = x$ and state the number of roots of the equation $e^{\frac{1}{5}x} = x$. [3]

   (ii) Use the Newton-Raphson method with $x_0 = 0$ to determine the value of a root of the equation $e^{\frac{1}{5}x} = x$ correct to 3 decimal places. [4]

6   The equation $x^3 - 5x + 3 = 0$ has a root between $x = 0$ and $x = 1$.

   (i) The equation can be rearranged into the form $x = g(x)$ where $g(x) = px^3 + q$. State the values of p and q. [1]

   (ii) By considering $|g'(x)|$, show that the iterative form $x_{n+1} = g(x_n)$ with a suitable starting value converges to the root between $x = 0$ and $x = 1$. [You are not required to f nd this root.] [2]