

Trigonometry

Question Paper

Level	Pre U
Subject	Maths
Exam Board	Cambridge International Examinations
Topic	Trigonometry
Booklet	Question Paper

Time Allowed: 122 minutes

Score: /102

Percentage: /100

Grade Boundaries:

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1 0 (i) Prove that $\cot \theta + \frac{\sin \theta}{1 + \cos \theta} = \operatorname{cosec} \theta$. [4]

(ii) Hence solve the equation $\cot\left(\theta + \frac{\pi}{4}\right) + \frac{\sin\left(\theta + \frac{\pi}{4}\right)}{1 + \cos\left(\theta + \frac{\pi}{4}\right)} = \frac{5}{2}$ for $0 \leq \theta \leq 2\pi$. [4]

2 Sketch the curve with equation $y = \tan x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

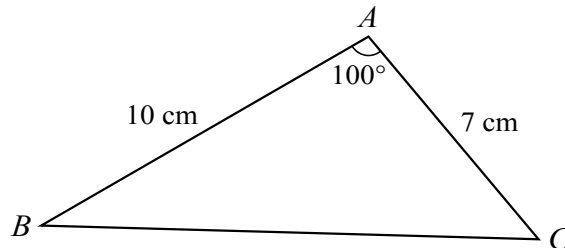
On the same diagram, sketch the curve with equation $y = \tan^{-1} x$ for all x . [3]

State the geometrical relationship between the curves.

3 (i) Use the identity $\tan 2x \equiv \frac{2 \tan x}{1 - \tan^2 x}$ to show that $\tan 4x \equiv \frac{4(1 - \tan^2 x) \tan x}{1 - 6 \tan^2 x + \tan^4 x}$. [6]

(ii) Hence, given that $x = \frac{1}{16}\pi$ is a root of the equation $\tan^4 x + p \tan^3 x - 6 \tan^2 x - p \tan x + 1 = 0$ where p is a positive constant, find the value of p . [4]

4 The diagram shows the triangle ABC . $AB = 10$ cm, $AC = 7$ cm and angle $BAC = 100^\circ$.



(i) Find the length BC . [2]

(ii) Find the area of the triangle ABC . [2]

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- 5 Given that the angle θ is acute and $\cos \theta = \frac{3}{4}$ find, without using a calculator, the exact value of $\sin 2\theta$ and of $\cot \theta$. [5]
- 6 (i) Sketch the graph of $y = \cos 2x$ for $0 \leq x \leq 2\pi$. [2]
(ii) Describe the transformation which maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. [3]
- 7 (i) Show that $\sin \theta + \sqrt{3} \cos \theta$ can be expressed in the form $R \sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [4]
State the values of R and α .
(ii) Hence find the value of θ , where $0 < \theta < \pi$, such that $\sin \theta + \sqrt{3} \cos \theta = 0.8$. [4]
- 8 (i) Prove that $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$ and hence find an exact value for $\tan\left(\frac{3}{8}\pi\right)$. [6]
(ii) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi} (\operatorname{cosec} 2x - \cot 2x)^2 dx$. [6]
- 9 (i) Prove that
$$\sin^2 2\theta(\cot^2 \theta - \tan^2 \theta) = 4(\cos^4 \theta - \sin^4 \theta)$$
and hence show that
$$\sin^2 2\theta(\cot^2 \theta - \tan^2 \theta) = 4 \cos 2\theta$$
. [5]
(ii) Hence solve the equation $\sin^2 2\theta(\cot^2 \theta - \tan^2 \theta) = 2$ for $0^\circ \leq \theta < 360^\circ$. [4]

- 10 Sketch, on separate diagrams, the graphs of the following functions for $0 \leq x \leq 2\pi$ giving the coordinates of all points of intersection with the axes.

(i) $y = \sin x$. [1]

(ii) $y = \sin\left(x + \frac{1}{6}\pi\right)$. [2]

- 11 (i) Prove that $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$ and deduce that

$$\sin \theta + \sin 3\theta = 4 \sin \theta \cos^2 \theta. \quad [5]$$

- (ii) Hence find the values of θ such that $0^\circ < \theta < 180^\circ$ that satisfy the equation

$$\cot^2 \theta = \sin \theta + \sin 3\theta. \quad [4]$$

- 12 (i) On the same diagram, sketch the graphs of $y = 2 \sec x$ and $y = 1 + 3 \cos x$, for $0 \leq x \leq \pi$. [4]

- (ii) Solve the equation $2 \sec x = 1 + 3 \cos x$, where $0 \leq x \leq \pi$. [5]

- 13 (i) Write down an identity for $\tan 2\theta$ in terms of $\tan \theta$ and use this result to show that

$$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}. \quad [4]$$

- (ii) Given that $0 < \theta < \frac{1}{2}\pi$ and $\theta = \sin^{-1}\left(\frac{1}{\sqrt{10}}\right)$, show that $\tan 3\theta = \frac{13}{9}$. [3]

- (iii) Show that the solutions of the equation

$$\tan(3 \sin^{-1} x) = \frac{13}{9}$$

for $0 < x < 2\pi$ are

$$x = \frac{\sqrt{10}}{10} \quad \text{and} \quad x = \frac{\sqrt{10}(1 + 3\sqrt{3})}{20}. \quad [4]$$

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14 (i) Show that

$$\cos^4 x - \sin^4 x \equiv 2 \cos^2 x - 1. \quad [2]$$

(ii) Hence find the solutions of

$$\cos^4 x - \sin^4 x = \cos x,$$

where $0^\circ \leq x \leq 360^\circ$.

[4]