PapaCambridge

A Level

Statistics – 1

PAST PAPERS

9709/5

CONTENTS

Representation of Data ---------------------- Pg 02

Probability ----------------------------------Pg 08

Permutations and Combinations ------------- Pg 12

DRV, Binomial & Geometric Distribution --------Pg 15

Normal Distribution -------------------------Pg 19

Representation of Data

Question - 1:

50 values of the variable x are summarised by

$$\Sigma(x - 20) = 35 \qquad \text{and} \qquad \Sigma x^2 = 25\,036.$$

Find the variance of these 50 values. [3]

Working:

$$\bar{x} = \frac{35}{50} + 20 = 20.7$$

$$\sigma^2 = \frac{25036}{50} - (20.7)^2$$

$$= 72.23$$

$$= 72.2 \text{ (3s.f)}$$

Marking Scheme:

Question	Answer	Marks	Guidance
1	$\sum x - 50\times 20 = 35$; $\sum x = 1035$ or $\bar{x} = \frac{35}{50} + 20 = \frac{1035}{50}$ [= 20.7]	**B1**	Correct value for $\sum x$ or $\bar{x}$.
	$\frac{25036}{50} - \left(\frac{\sum x}{50}\right)^2 = \frac{25036}{50} - \left(\frac{1035}{50}\right)^2$	**M1**	$\frac{25036}{50} - \left(\textit{their}\left(\frac{\sum x}{50}\right)^2\right)$
	72.23	**A1**	Exact answer only **SC B1** for 72.23 with no substitution in formula.
		3	

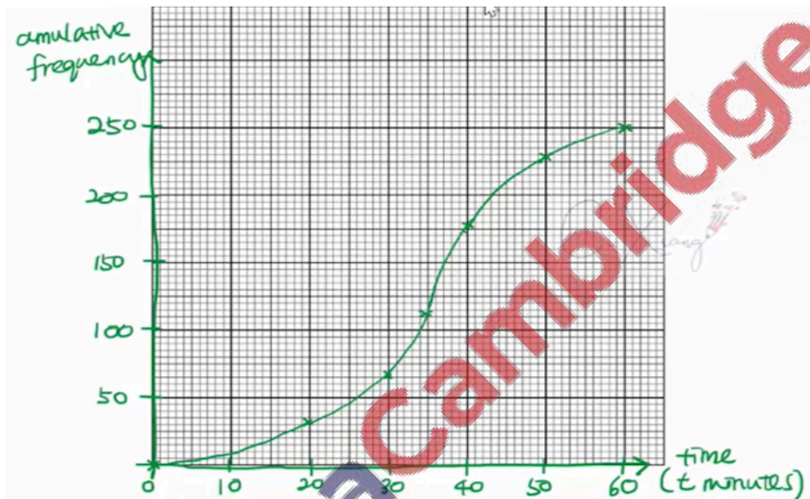
Question - 2:

The times, t minutes, taken to complete a walking challenge by 250 members of a club are summarised in the table.

Time taken (t minutes)	$t \leqslant 20$	$t \leqslant 30$	$t \leqslant 35$	$t \leqslant 40$	$t \leqslant 50$	$t \leqslant 60$
Cumulative frequency	32	66	112	178	228	250

(a) Draw a cumulative frequency graph to illustrate the data. [2]

Solution:



(b) Use your graph to estimate the 60th percentile of the data. [1]

Solution:

$$\frac{250 \times 60}{100} = 150$$

$t = 37$ minutes

Part (C):

It is given that an estimate for the mean time taken to complete the challenge by these 250 members is 34.4 minutes.

(c) Calculate an estimate for the standard deviation of the times taken to complete the challenge by these 250 members. [4]

Solution:

Class	0-20	20-30	30-35	35-40	40-50	50-60
x	10	25	32.5	37.5	45	55
f	32	34	46	66	50	22

$$\Sigma f x^2 = 32(10^2) + 34(25)^2 + 46(32.5)^2 + 66(37.5)^2 + 50(45)^2 + 22(55)^2$$

$$= 33650$$

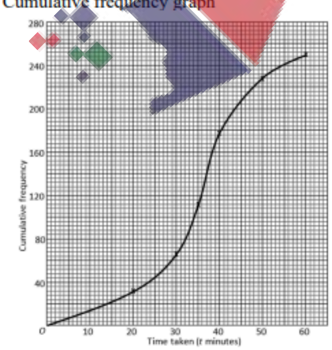
$$\sigma = \sqrt{\frac{33650}{250} - (34.4)^2} = \sqrt{151.24}$$

$$= 12.3$$

Marking Scheme:

Question	Answer	Marks	Guidance
3(c)	[Frequencies] [32] 34 46 66 50 22	B1	May be unsimplified and/or in variance calculation.
	[Midpoints] 10 25 32.5 37.5 45 55	M1	At least 5 correct midpoints seen , may be unsimplified.
	[Variance] = $\frac{32\times10^2+34\times25^2+46\times32.5^2+66\times37.5^2+50\times45^2+22\times55^2}{250}-34.4^2$ [$=\frac{333650}{250}-34.4^2=151.24$]	M1	Correct unsimplified Variance formula with *their* midpoints and *their* frequencies for var or sd. (− mean² included)
	[Sd =] 12.3	A1	Awrt WWW **SC B1** for 12.3 if second M1 not awarded.
		4	

Marking Scheme:

Question	Answer	Marks	Guidance
3(a)	Cumulative frequency graph 	M1	At least 3 points plotted accurately at class upper end points: (20,32), (30, 66), (35, 112), (40, 178), (50, 228), (60, 250). Linear cf scale 0 ⩽ cf ⩽ 250 and linear time scale 0 ⩽ time ⩽ 60 with at least 3 values identified on each.
		A1	All points plotted correct, curve drawn (within tolerance) and joined to (0,0). Axes labelled cumulative frequency (cf), time (t) and minutes (min or m) – or a suitable title. Axes can be the other way round.
		2	
3(b)	Line drawn from 150 on cf axis to meet graph at about t =38 minutes	B1 FT	Must be an increasing cf graph with correct upper bounds. Use of graph must be seen. Expect an answer in range 37 ⩽ t ⩽ 39 for a correct graph
		1	

Question - 3:

For n values of the variable x, it is given that

$$\Sigma(x - 200) = 446 \quad \text{and} \quad \Sigma x = 6846.$$

Find the value of n. [3]

Solution:

$$\Sigma\left(\frac{x-200}{n}\right) + 200 = \bar{x}$$

$$\Sigma\left(\frac{x-200}{n}\right) + 200 = \frac{\Sigma x}{n}$$

$$\frac{446}{n} + 200 = \frac{6846}{n}$$

$$\Rightarrow \frac{6846}{n} - \frac{446}{n} = 200$$

$$\frac{6400}{n} = 200$$

$$n = \frac{6400}{200}$$

$$n = 32$$

Marking Scheme:

Question	Answer	Marks	Guidance
1	$\sum x - \sum 200 = \sum(x - 200)$	B1	Forming a correct 3-term (linear) equation from $\sum x$, $\sum 200$ and $\sum(x-200)$. Accept $6846 - 200n = 446$ OE. Condone 1 sign error.
	$\sum 200 = 200n$	B1	SOI
	$[200n = 6846 - 446 = 6400]$ $n = 32$	B1	WWW
		3	

Question - 4:

The back-to-back stem-and-leaf diagram shows the diameters, in cm, of 19 cylindrical pipes produced by each of two companies, A and B.

Company A							Company B				
					4	33	1	2	8		
	9	8	3	2	0	34	1	6	8	9	9
8	7	5	4	1	1	35	1	2	2	3	
		9	6	5	2	36	5	6			
			4	3	1	37	0	3	4		
						38	2	8			

Key: 1 | 35 | 3 means the pipe diameter from company A is 0.351 cm and from company B is 0.353 cm.

(a) Find the median and interquartile range of the pipes produced by company A. [3]

Solution:

$$\text{Median} = \frac{19+2}{2} = 10^{th} \text{ Value}$$

$$\text{Median} = 0.355 \text{ cm}$$

$$\text{IQR} = 0.366 - 0.348$$

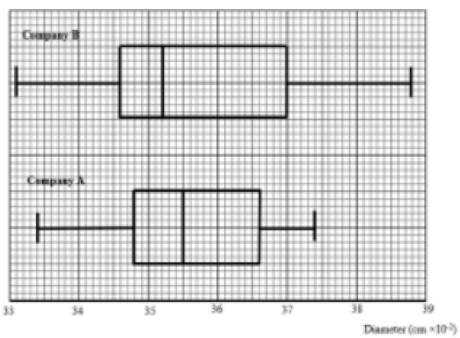
$$= 0.018 \text{ cm}$$

It is given that for the pipes produced by company B the lower quartile, median and upper quartile are 0.346 cm, 0.352 cm and 0.370 cm respectively.

(b) Draw box-and-whisker plots for companies A and B on the grid below. [3]

(c) Make one comparison between the diameters of the pipes produced by companies A and B. [1]

Marking Scheme:

Question	Answer	Marks	Guidance
3(a)	Median = 0.355	**B1**	Identified condone Q2.
	[IQR =] 0.366 – 0.348	**M1**	0.365 ⩽ UQ ⩽ 0.369 – 0.343 ⩽ LQ ⩽ 0.349. Subtraction may be implied by answer.
	0.018	**A1**	If 0/3 scored **SC B1** for figs Median = 355 IQR = 18.
		3	
3(b)	Box-and-whisker plot on provided grid 	**B1**	All 5 key values for *B* plotted accurately in standard format using *their* scale. Labelled *B*. Check accuracy in the middle of vertical line.
		B1 FT	All 5 key values for *A*, FT from part **3(a),** plotted in standard format accurately using *their* scale. Labelled *A*. Check accuracy in the middle of vertical line.
		B1	Whiskers not through box for both, not drawn at corners of boxes, single linear scale with at least 3 values stated, covering at least 0.34 to 0.38 and labelled diameter (*d* etc) and cm. Accept as a title.
		3	If both plots attempted and plot(s) not labelled, **SC B1** for at least 1 fully correct set of values plotted.
3(c)	A comparison in context	**B1**	Single comment comparing spread or central tendency in context. Must reference either diameter or pipes. Not a simple numerical comparison of statistical values such as median, range, IQR or min/max.
		1	

Probability

Question - 1:

Three fair 4-sided spinners each have sides labelled 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which each spinner lands is recorded. The random variable X denotes the highest number recorded.

(a) Show that $P(X = 2) = \frac{7}{64}$. [3]

Solution:

$(1,1,2) \times 3$
$(1,2,2) \times 3$
$(2,2,2)$

$P(x = 2) = P(1,1,2) \times 3 + P(1,2,2) \times 3 + P(2,2,2)$

$= \frac{1}{4}\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) \times 3 + \frac{1}{4}\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) \times 3 + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$

$= \frac{7}{64}$ (shown)

Complete the probability distribution table for X. [2]

x	1	2	3	4
$P(X = x)$		$\frac{7}{64}$	$\frac{19}{64}$	

Solution:

$P(x = 1) = P(1,1,1)$

$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{64}$

$P(x = 4) = 1 - \frac{1}{64} - \frac{7}{64} - \frac{19}{64}$

$= \frac{37}{64}$

Marking Scheme:

Question	Answer	Marks	Guidance
4(a)	**Method 1: Scenarios identified**		
	[no of ways for score of 2 are] 222, 211, 212, 221, 122, 112, 121 [Total options = 64]	B1	7 correct scenarios identified, no incorrect.
	[So P(X = 2) =] $\frac{7}{4\times4\times4} = \frac{7}{64}$	M1	$\frac{a}{4\times4\times4}$, a = *their* number of correct identified scenarios > 4
		A1	Approach identified, WWW.
	Method 2: P(2 on all spinners) + P(2 on two spinners and 1 on one spinner) + P(2 on one spinner and 1 on two spinners)		
	$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + {}^3C_1\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)$	B1	$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(or\ {}^3C_1\right)\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + d$, $0 < d < 1$
		M1	$\left(\frac{1}{4}\right)^3 + e\left(\frac{1}{4}\right)^3 + f\left(\frac{1}{4}\right)^3$ $1 < e < 5$ and $1 < f < 5$
	[So P(X = 2) =] = $\frac{7}{64}$	A1	Approach identified, WWW.

Question	Answer	Marks	Guidance
	Method 3: P(1 or 2 on each spinner) – P(1 on all spinners)		
	$\left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3$	B1	$\left(\frac{1}{2}\right)^3 - b$ seen, $0 < b < 1$
		M1	$\left(\frac{1}{2}\right)^3 - c^3$, $0 < c < ½$
	[So P(X = 2) =] $\frac{7}{64}$	A1	Approach identified, WWW.
		3	
4(b)	P(X = 1) = $\frac{1}{64}$	B1	P(X = 1) or P(X = 4) correct. Condone answers not in probability distribution table if clearly identified.
	P(X = 4) = $\left[1 - \frac{1}{64} - \frac{7}{64} - \frac{19}{64} = \right]\frac{37}{64}$	B1 FT	All 4 probabilities summing to 1.
		2	

Question - 2:

A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the probability distribution table for X. [3]

Solution:

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

x	2	3	4	5	6
$P(X=x)$	$1/36$	$4/36$	$10/36$	$12/36$	$9/36$

Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

Solution:

$$E(X) = \left(2 \times \frac{1}{36}\right) + \left(3 \times \frac{4}{36}\right) + \left(4 \times \frac{10}{36}\right) + \left(5 \times \frac{12}{36}\right) + \left(6 \times \frac{9}{36}\right)$$

$$= \frac{2+12+40+60+54}{36} = \frac{168}{36}$$

$$\mathrm{Var}(X) = \Sigma x^2 p - (E(x))^2$$

$$= \left[\left(2^2 \times \frac{1}{36}\right) + \left(3^2 \times \frac{4}{36}\right) + \left(5^2 \times \frac{12}{36}\right) + \left(6^2 \times \frac{9}{36}\right) + \left(4^2 \times \frac{10}{36}\right) - \left(\frac{168}{36}\right)^2\right]$$

$$= \frac{10}{9} \text{ or } 1.11$$

Marking Scheme:

Question	Answer	Marks	Guidance
2(a)	x: 2, 3, 4, 5, 6 p: $\frac{1}{36}$, $\frac{4}{36}$, $\frac{10}{36}$, $\frac{12}{36}$, $\frac{9}{36}$ 0.02778, 0.1111, 0.2778, 0.3333, 0.25	B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
		B1	3 correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		B1	2 further correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		3	**SC B1** for 5 probabilities ($0 < p < 1$) that sum to 1 with less than 3 correct probabilities.

Question	Answer	Marks	Guidance
2(b)	If method FT from *their* incorrect (a), expressions for E(X) and Var(X) must be seen at the stage shown in **bold** (or less simplified) in the scheme with all probabilities < 1.		
	$\left[\mathrm{E}(X) = \frac{1\times2+4\times3+10\times4+12\times5+9\times6}{36} =\right]$ $\mathbf{\frac{2+12+40+60+54}{36}}$	M1	Accept unsimplified expression. May be calculated in variance. FT *their* table with 4 or more probabilities summing to $0.999 \leqslant$ total $\leqslant 1$ $(0 < p < 1)$.
	$\left[\mathrm{Var}(X) = \frac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36} - (their\ \mathrm{E}(X))^2 =\right]$ $\mathbf{\frac{1\times4+4\times9+10\times16+12\times25+9\times36}{36}} - \left(their\ \mathbf{\frac{14}{3}}\right)^2$ $\left[\frac{4+36+160+300+324}{36} - \left(their\ \frac{14}{3}\right)^2\right]$	M1	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 3 or more probabilities $(0 < p < 1)$ which need not sum to 1 and the calculation in **bold** (or less simplified) seen.
	$\mathrm{E}(X) = \frac{168}{36}, \frac{14}{3}, 4.67$ $\mathrm{Var}(X) = \frac{10}{9}, 1\frac{1}{9}, 1.11, \frac{1440}{1296}$	A1	Answers for E(X) and Var(X) must be identified. E(X) may be identified by correct use in Variance. Condone E, V, μ, σ^2 etc. If M0 earned **SC B1** for identified correct final answers.
		3	

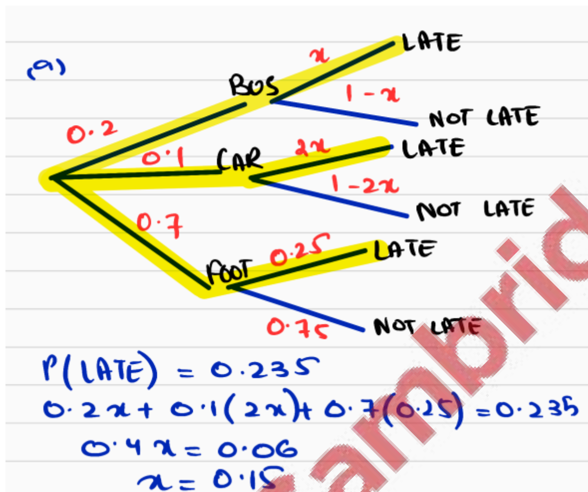
Question - 3:

On any day, Kino travels to school by bus, by car or on foot with probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25.

The probability that, on a randomly chosen day, Kino is late is 0.235.

(a) Find the value of x. [3]

Solution:



Marking Scheme:

Question	Answer	Marks	Guidance
1(a)	$0.2 \times x + 0.1 \times 2x + 0.7 \times 0.25 = 0.235$	M1	$0.2 \times x + 0.1 \times 2x + 0.7 \times 0.25$ or $0.2x + 0.2x + 0.175$ seen.
		M1	Equating *their* 3 term expression (2 terms involving x) to 0.235
	$x = 0.15$	A1	
		3	

Permutations and Combinations

Question - 1:

(a) Find the number of different arrangements of the 9 letters in the word ACTIVATED. [2]

Solution:

$$\frac{9!}{2!\,2!} = 90720$$

Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. [3]

Solution:

I : A _ _ _ _ _ A _ _ $\quad \frac{7!}{2!} \times 3 = 7560$

_ A _ _ _ _ _ A _

_ _ A _ _ _ _ _ A

II : A _ _ _ _ _ _ A _ $\quad \frac{7!}{2!} \times 2 = 5040$

_ A _ _ _ _ _ _ A

III : A _ _ _ _ _ _ _ A $\quad \frac{7!}{2!} = 2520$

Total = 7560 + 5040 + 2520

= 15120

Marking Scheme:

Question	Answer	Marks	Guidance
6(a)	$\frac{9!}{2!2!}$	M1	$\frac{h!}{2! \times j!}$, $h = 7, 8, 9; j = 1, 2$
	90720	A1	
		2	

6(b)	**Arrangements with 5 letters between As + Arrangements with 6 letters between As + Arrangements with 7 letters between As**		
	With gap of 5: $\frac{7!}{2!}\times 3$ [= 7560] With gap of 6: $\frac{7!}{2!}\times 2$ [= 5040] With gap of 7: $\frac{7!}{2!}\times 1$ [= 2520]	M1	$\frac{7!}{2!}\times k$, k positive integer $1 < k < 7$
		M1	Add their no of ways for 3 identified correct scenarios, no additional incorrect scenarios, accept unsimplified.
	[Total no = $\frac{7!}{2!}\times 6 =$] 15120	A1	
		3	

Question - 2:

(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

Solution:

CROCODILE

C C OO ROILE

$\frac{9!}{2!\,2!}$ = 90720 arrangements

Marking Scheme:

Question	Answer	Marks	Guidance
6(a)	$\left[\frac{9!}{2!2!}=\right]$ 90 720	B1	
		1	

Question - 3:

(a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

Solution:

2A 2L 1I 1G 1T 1O 1R

AA LL I G T O R

$7!$ = 5040 arrangements.

Marking Scheme:

Question	Answer	Marks	Guidance
7(a)	7!	M1	$\frac{7!}{b! \times c!}$ $b, c = 1, 2$ $7! \times \frac{2!}{2!} \times \frac{2!}{2!}$ oe, no further terms present.
	5040	A1	
		2	

Question - 4

The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

Solution:

without restrictions $= \frac{9!}{2!2!} = 90720$

A _ _ _ _ _ _ A _ $2 \times 5 \times 5! = 1200$

_ A _ _ _ _ _ _ A

for 6 letters b/w A

for L must be together

remaining avg: 5 diff. letters

prob $= \frac{1200}{90720} = \frac{5}{378}$ or 0.0132

Marking Scheme:

Question	Answer	Marks	Guidance
7(b)	**Method 1 for first 3 marks: Arrangements of 6 letters including Ls between As**		
	$5! \times 5 \times 2$	M1	$5! \times d$, d integer > 1
		M1	$e! \times f \times g$, $e = 5, 6, 7$; $f = 1, 5$; $g = 1, 2$; $f \neq g$. 1 can be implicit.
	1200	A1	
	Method 2 for first 3 marks: Number of arrangements of LL^^^^^ – number of arrangements with the Ls split by an A		
	$6! \times 2 - 5! \times 2$	M1	$6! \times 2 - h$ h an integer $1 < h < 1440$
		M1	$k - 5! \times 2$ k an integer $k > 240$
	1200	A1	
	Method 3 for first 3 marks: Alternative approaches to Method 1		
	^A ^ ^ ^ ^ ^ ^ A $^5P_1 \times {}^1P_1 \times {}^5P_5 \times {}^1P_1 = 600$	M1	LL treated as a single unit.
		M1	
	1200	A1	

Question	Answer	Marks	Guidance
7(b)	**Final 2 marks of Question 7(b)**		
	[Total number of arrangements =] $\left[\frac{9!}{2!2!} = \right] 90720$	B1	Accept unsimplified. May be seen as denominator of probability.
	Probability $= \frac{1200}{90720}, \frac{5}{378}, 0.0132$	B1 FT	$\frac{\textit{their } 1200}{\textit{their } 90720}$ unsimplified B1 FT if *their* 1200 and *their* 90 720 supported by work in this part.
		5	

DRV, Binomial and Geometric Distribution

Question - 1:

On another occasion, one of the fair 4-sided spinners is spun repeatedly until a 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

(c) Find $P(Y = 6)$. [1]

Solution:

$$P(Y=6) = \left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)$$

$$= 0.059326171$$

$$= 0.0593 \ (3sf)$$

$$\boxed{\begin{array}{l} Y \sim Geo(p) \\ P(Y=r) = q^{r-1}\,p \end{array}}$$

(d) Find $P(Y > 4)$. [2]

Solution:

$$P(Y>4) = 1 - P(Y \leq 4)$$

$$= 1 - \left[\frac{1}{4} - \left(\frac{3}{4}\right)^4\right]$$

$$= \left(\frac{3}{4}\right)^4$$

$$= \frac{81}{256} \quad \text{or } 0.316$$

Marking Scheme:

4(c)	$P(Y=6) = \left[\left(\frac{3}{4}\right)^5 \times \frac{1}{4} = \right] 0.0593, \frac{243}{4096}$	B1	Accept 0.059326… to 4 or more SF.

Question	Answer	Marks	Guidance
4(d)	$\left(\frac{3}{4}\right)^4$	M1	$\left(\frac{3}{4}\right)^g$, $g = 4, 5$ **or** p^4 where $0 < p < 1$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406…to 4 or more SF.
	Alternative method for Question 4(d)		
	$P(Y>4) = 1 - P(Y \leqslant 4) = 1 - \left(\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4}\right)$ $\left[= 1 - \frac{175}{256}\right]$	M1	Correct or $1 - \left(\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \left(\frac{3}{4}\right)^2 \times \frac{1}{4} + \left(\frac{3}{4}\right)^3 \times \frac{1}{4} + \left(\frac{3}{4}\right)^4\right)$ **or** $1 - (p + qp + q^2p + q^3p)$ where $0 < p < 1$ and $q = 1 - p$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406…to 4 or more SF.
		2	

Question - 2:

Company *A* produces bags of sugar. An inspector finds that on average 10% of the bags are underweight.

10 of the bags are chosen at random.

(a) Find the probability that fewer than 3 of these bags are underweight. [3]

Solution:

$X \sim B(10, 0.10)$

$P(X < 3) = {}^{10}C_0 \times 0.1^0 \times 0.9^{10} +$
${}^{10}C_1 \times 0.1^1 \times 0.9^9 +$
${}^{10}C_2 \times 0.1^2 \times 0.9^8$
$= 0.929809$
$= 0.930 \ (3sf)$

Marking Scheme:

Question	Answer	Marks	Guidance
5(a)	[P(0, 1, 2) =] ${}^{10}C_0\ 0.1^0\,0.9^{10} + {}^{10}C_1\ 0.1^1\,0.9^9 + {}^{10}C_2\ 0.1^2\,0.9^8$	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1, x \neq 0$
	= 0.348678+0.38742+0.19371	A1	Correct expression, accept unsimplified.
	0.930	B1	$0.9298 \leqslant p \leqslant 0.9303$
	Alternative method for Question 5(a)		
	[1 – P(3, 4, 5, 6, 7, 8, 9, 10) = 1 – (${}^{10}C_3\ 0.9^7\,0.1^3 + {}^{10}C_4\ 0.9^6\,0.1^4 + {}^{10}C_5\ 0.9^5\,0.1^5 + {}^{10}C_6\ 0.9^4\,0.1^6 + {}^{10}C_7\,0.9^3\,0.1^7 + {}^{10}C_8\ 0.9^2\,0.1^8 + {}^{10}C_9\ 0.9^1\,0.1^9 + {}^{10}C_{10}\ 0.9^0\,0.1^{10}$)	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1, x \neq 0$
		A1	Correct expression, accept unsimplified.
	0.930	B1	$0.9298 \leqslant p \leqslant 0.9303$
		3	

Question - 3:

A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

Find $E(X)$ and $Var(X)$. [3]

Probability distribution table for the given question above is already solved/given.

x	2	3	4	5	6
$P(X=x)$	$1/36$	$4/36$	$10/36$	$12/36$	$9/36$

Solution:

$$E(X) = \left(2 \times \frac{1}{36}\right) + \left(3 \times \frac{4}{36}\right) + \left(4 \times \frac{10}{36}\right) + \left(5 \times \frac{12}{36}\right) + \left(6 \times \frac{9}{36}\right)$$

$$= \frac{2+12+40+60+54}{36} = \frac{168}{36}$$

$$Var(X) = \sum x^2 p - (E(x))^2$$

$$= \left[\left(2^2 \times \frac{1}{36}\right) + \left(3^2 \times \frac{4}{36}\right) + \left(5^2 \times \frac{12}{36}\right) + \left(6^2 \times \frac{9}{36}\right) + \left(4^2 \times \frac{10}{36}\right) - \left(\frac{168}{36}\right)^2\right]$$

$$= \frac{10}{9} \text{ or } 1.11$$

Question - 4:

In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

Solution:

$X \sim B(n, p)$

$X \sim B(12, 0.72)$

$P(X > 9) = P(X = 10, 11, 12)$

${}^{12}C_{10}(0.72)^{10}(0.28)^2 + {}^{12}C_{11}(0.72)^{11}(0.28)^1$

$+ {}^{12}C_{12}(0.72)^{12}(0.28)^0$

$= 0.304$

Marking Scheme:

Question	Answer	Marks	Guidance
5(a)	[P(10, 11, 12) =] ${}^{12}C_{10}\,0.72^{10}\,0.28^2 + {}^{12}C_{11}\,0.72^{11}\,0.28^1 + {}^{12}C_{12}\,0.72^{12}\,0.28^0$	M1	One term ${}^{12}C_x\, p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
	$= 0.193725 + 0.0905726 + 0.0194084$	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
	Alternative method for question 5(a)		
	[1 – P(0,1,2,3,4,5,6,7,8,9) =] $1 - ({}^{12}C_0\,0.72^0\,0.28^{12} + {}^{12}C_1\,0.72^1\,0.28^{11} + {}^{12}C_2\,0.72^2\,0.28^{10} + {}^{12}C_3\,0.72^3\,0.28^9 + {}^{12}C_4\,0.72^4\,0.28^8 + {}^{12}C_5\,0.72^5\,0.28^7 + {}^{12}C_6\,0.72^6\,0.28^6 + {}^{12}C_7\,0.72^7\,0.28^5 + {}^{12}C_8\,0.72^8\,0.28^4 + {}^{12}C_9\,0.72^9\,0.28^3)$	M1	One term ${}^{12}C_x\, p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
		3	

Normal Distribution

Question - 1:

In a large college, 32% of the students have blue eyes. A random sample of 80 students is chosen.

Use an approximation to find the probability that fewer than 20 of these students have blue eyes. [5]

Solution:

$X \sim B(80, 0.32)$

$X \sim N(\underset{np}{25.6}, \underset{npq}{17.408})$

$$P(X < 20) = P(X < 19.5)$$
$$= P\left(Z < \frac{19.5 - 25.6}{\sqrt{17.408}}\right)$$
$$= P(Z < -1.462)$$
$$= 0.0718$$

Marking Scheme:

Question	Answer	Marks	Guidance
2	Mean = $80 \times 0.32 = 25.6$, var $= 80 \times 0.32 \times 0.68 = 17.408$	B1	25.6 and 17.4[08] seen, allow unsimplified. 4.172… implies correct variance.
	$P(X < 20) = P(Z < \frac{19.5 - 25.6}{\sqrt{17.408}}) = P(Z < -1.462)$	M1	Substituting *their* 25.6 and 17.408 into ±standardisation formula (any number for 19·5), not σ^2, $\sqrt{\sigma}$.
		M1	Using continuity correction 19·5 or 20·5 in *their* standardisation formula.
	$= [1 - \Phi(1.462)] = 1 - 0.9282$	M1	Appropriate area Φ, from final process, must be probability. (Expect final ans < 0·5). Note: the correct final answer may imply M1 from use of calculator.
	0.0718	A1	$0.0718 \leqslant p \leqslant 0.0719$
		5	

Question - 2 and 3:

The weights of the bags of sugar produced by company B are normally distributed with mean 1.04 kg and standard deviation 0.06 kg.

(b) Find the probability that a randomly chosen bag produced by company B weighs more than 1.11 kg. [3]

Solution:

$X \sim N(1.04, 0.06^2)$

$P(X > 1.11) = P\left(Z > \frac{1.11 - 1.04}{0.06}\right)$

$= P(Z > 1.167)$

$= 1 - 0.8784$

$= 0.122 \text{ (3sf)}$

Marking Scheme:

5(b)	$[P(X > 1.11) =]P(Z > \frac{1.11 - 1.04}{0.06}) = P(Z > 1.167)$	M1	1.11, 1.04 and 0.06 substituted into ±Standardisation formula, no continuity correction not 0.06^2 or $\sqrt{0.06}$
	$= 1 - 0.8784$	M1	1 – *their* 0.8784 as final answer, must be probability. (Expect final ans < 0·5).
	0.122	A1	$0.1216 \leqslant p \leqslant 0.122$ **SC M0 M1 B1** for 0.122 with no standardisation formula.
		3	

81% of the bags of sugar produced by company B weigh less than w kg.

(c) Find the value of w. [3]

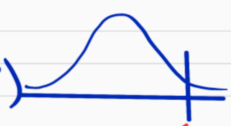
Solution:

$P(X < w) = 0.81$

$P\left(Z < \frac{w - 1.04}{0.06}\right) = 0.81$

$\frac{w - 1.04}{0.06} = 0.878$

$w = 1.09 \text{ (3.sf)}$



$\frac{w - 1.04}{0.06}$

$z = 0.878$

Marking Scheme:

Question	Answer	Marks	Guidance
5(c)	$[P(X < w) = P(Z < \frac{w - 1.04}{0.06}) = 0.81]$ $\frac{w - 1.04}{0.06} = 0.878$	B1	$0.8775 < z \leqslant 0.878$ or $-0.878 \leqslant z < -0.8775$ seen.
		M1	1.04 and 0.06 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$w = 1.09$	A1	$1.09 \leqslant w \leqslant 1.093$
		3	

Question - 4:

The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

(a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

Solution:

$X \sim N(2.02, 0.03^2)$

$P(1.98 < x < 2.03)$

$P(x < 2.03) - P(x < 1.98)$

$P\left(z < \frac{2.03 - 2.02}{0.03}\right) - P\left(z < \frac{1.98 - 2.02}{0.03}\right)$

$P(z < 0.333) - P(z < -1.33)$

$P(z < 0.333) - [1 - P(z < 1.33)]$

$= 0.6304 - 1 + 0.9087$

$= 0.5391$

$= 0.539$

Marking Scheme:

Question	Answer	Marks	Guidance
4(a)	$[P(1.98 < X < 2.03) =]P\left(\frac{1.98-2.02}{0.03} < z < \frac{2.03-2.02}{0.03}\right)$ $[= P(-1.333 < z < 0.333)]$	M1	Use of ±standardisation formula once with 2.02, 0.03 and either 1.98 or 2.03 substituted appropriately. Condone 0.03^2 and continuity correction ±0.005, not $\sqrt{0.03}$.
	$[= \Phi(0.333) - (1 - \Phi(1.333))]$ $= 0.6304 + 0.9087 - 1$	M1	Calculating the appropriate probability area from *their* z-values. (or $0.6304 - 0.09121$ or $(0.9087 - 0.5) + (0.6304 - 0.5)$ etc)
	0.539	A1	$0.539 \leqslant z < 0.5395$ Only dependent upon 2nd M mark. If M0 scored **SC B1** for $0.539 \leqslant z < 0.5395$.
		3	