



Cambridge International AS & A Level

CANDIDATE NAME



CENTRE NUMBER

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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

October/November 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.





1 The matrix \mathbf{M} represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor k ($k \neq 0$), followed by a shear, x -axis fixed, with $(0, 1)$ mapped to $(k, 1)$.

(a) Show that $\mathbf{M} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$. [4]

Dotted lines for answer (a)

(b) The transformation represented by \mathbf{M} has a line of invariant points.

Find, in terms of k , the equation of this line. [3]

Dotted lines for answer (b)





The unit square S in the x - y plane is transformed by \mathbf{M} onto the parallelogram P .

(c) Find, in terms of k , a matrix which transforms P onto S . [1]

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(d) Given that the area of P is $3k^2$ units², find the possible values of k . [2]

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2 Prove by mathematical induction that, for all positive integers n ,

$$\frac{d^n}{dx^n}(\tan^{-1}x) = P_n(x)(1+x^2)^{-n},$$

where $P_n(x)$ is a polynomial of degree $n - 1$.

[6]

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3 The quartic equation $x^4 + 2x^3 - 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are $\alpha^4, \beta^4, \gamma^4, \delta^4$ and state the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [5]

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(b) Find the value of $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$.

[3]

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(c) Find the value of $\alpha^8 + \beta^8 + \gamma^8 + \delta^8$.

[2]

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It is given that $\sum_{r=1}^{\infty} \frac{5k}{(5r+k)(5r+5+k)} = \frac{1}{3}$.

(b) Find the value of k . [2]

Dotted lines for writing the answer to part (b).

(c) Hence find $\sum_{r=n}^{n^2} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of n . [2]

Dotted lines for writing the answer to part (c).

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5 (a) Show that the curve with Cartesian equation

$$(x^2 + y^2)^2 = 6xy$$

has polar equation $r^2 = 3 \sin 2\theta$. [2]

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The curve C has polar equation $r^2 = 3 \sin 2\theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

(b) Sketch C and state the maximum distance of a point on C from the pole. [3]

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(c) Find the area of the region enclosed by C . [2]

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(d) Find the maximum distance of a point on C from the initial line. [6]

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(c) Sketch C , stating the coordinates of any intersections with the axes.

[5]

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(d) Sketch the curve with equation $y = \left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right|$ and state the set of values of k for which $\left| \frac{4x^2 + x + 1}{2x^2 - 7x + 3} \right| = k$ has 4 distinct real solutions. [2]



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7 The lines l_1 and l_2 have equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively. The plane Π_1 contains l_1 and is parallel to l_2 .

(a) Find the equation of Π_1 , giving your answer in the form $ax + by + cz = d$. [4]

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The plane Π_2 contains l_2 and the point with coordinates $(2, -1, 7)$.

(b) Find the acute angle between Π_1 and Π_2 . [4]

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The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 .

(c) Find a vector equation for PQ .

[7]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

Dotted lines for writing

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