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CAMBRIDGE INTERNATIONAL MATHEMATICS

0607/51

Paper 5 Investigation (Core)

May/June 2020

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

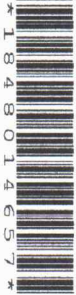
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

This document has 8 pages. Blank pages are indicated.



Answer **all** the questions.

COMBINING TRIANGLE NUMBERS

This investigation looks at results when adding or subtracting triangle numbers.

Here is a table of the first 21 triangle numbers, T_1 to T_{21} .

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}	T_{12}	T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}	T_{19}	T_{20}	T_{21}
1	3	6	10	15	21	28	36	45	55	66	78	91	105	120	136	153	171	190	210	231

- 1 Find the next two triangle numbers.

$$231 + 22 = 253$$

$$T_{22} = \dots\dots\dots 253$$

$$253 + 23 = 276$$

$$T_{23} = \dots\dots\dots 276 \quad [4]$$

- 2 (a) Complete the table.

T_1	1
$T_2 - T_1$	2
$T_3 - T_2$	3
$T_4 - T_3$	4
$T_5 - T_4$	5
$T_6 - T_5$	6
$T_n - T_{n-1}$	n

[2]

- (b) (i) $T_n - T_{n-1} = 100$.

Write down the value of n .

$$\dots\dots\dots 100 \quad [1]$$

- (ii) Write down the difference between the 50th and the 49th triangle numbers.

$$\dots\dots\dots 50 \quad [1]$$

- 3 Complete the table for adding two consecutive triangle numbers.

T_1	1
$T_2 + T_1$	4
$T_3 + T_2$	9
$T_4 + T_3$	16
$T_5 + T_4$	25
$T_6 + T_5$	36
$T_n + T_{n-1}$	n^2

[2]

- 4 (a) Use the last row of the table in Question 2(a) to complete the equation $T_n - T_{n-1} = \dots\dots\dots n \dots\dots$
 Use the last row of the table in Question 3 to complete the equation $T_n + T_{n-1} = \dots\dots\dots n^2 \dots\dots$
 By adding these two equations together show that $T_n = \frac{n^2 + n}{2}$.

$$2T_n = n^2 + n$$

$$T_n = \frac{n^2 + n}{2}$$

[1]

- (b) Find T_{1000}

$$\frac{1000^2 + 1000}{2}$$

$$= \frac{1000000 + 1000}{2}$$

$$= \frac{1000000}{2} + \frac{1000}{2}$$

$$= 500000 + 500$$

$$\dots\dots\dots 500500 \dots\dots\dots [2]$$

- 5 (a) The table shows the difference of the squares of two consecutive triangle numbers. Complete the table.

$(T_1)^2$	1
$(T_2)^2 - (T_1)^2$	8
$(T_3)^2 - (T_2)^2$	27
$(T_4)^2 - (T_3)^2$	64
$(T_5)^2 - (T_4)^2$	125
$(T_6)^2 - (T_5)^2$	216
$(T_n)^2 - (T_{n-1})^2$	n^3

[3]

- (b) Calculate the difference between the squares of the 50th and the 49th triangle numbers.

50³

12 500

[2]

- 6 The sum of two **different** triangle numbers sometimes equals another triangle number. When this happens, we have a *triangle triple*.

Example

- Start with the triangle number $T_3 = 6$.
- From the table in question 2(a) $T_6 - T_5 = 6$.
So $T_6 - T_5 = T_3$.
- Rearrange the equation $T_3 + T_5 = T_6$.
- The *triangle triple* is then (3, 5, 6).

The three different numbers must be written in order of increasing size.

- (a) Start with triangle number $T_5 = 15$ and complete the method of the Example to find another triangle triple.

$$T_{15} - T_{14} = 15$$

$$\text{So } T_{15} - T_{14} = T_5$$

$$T_5 + T_{14} = T_{15}$$

The triangle triple is (5, 14, 15)

[4]

- (b) In the table, each row is a triangle triple.
Use your answer to **part (a)** and any patterns you notice to complete the table.

Triangle triple		
3	5	6
4	9	10
5	14	15
6	20	21
7	27	28

[5]

- (c) Use the list of triangle numbers on page 2 to check the triangle triple beginning with 6.

$$21 + 210 = 231$$

[1]

- 7 (a) The triangle numbers T_1 and T_3 are not consecutive. They are two apart. Complete the table for subtracting triangle numbers that are two apart.

$T_3 - T_1$	5
$T_4 - T_2$	7
$T_5 - T_3$	9
$T_6 - T_4$	11
$T_7 - T_5$	13
$T_n - T_{n-2}$	$2n - 1$

[4]

- (b) Use the triangle number $T_9 = 45$ to find a triangle triple where

- the smallest number is 9
- the difference between the other two numbers is 2.

Hints: Use the last row of the table in **part (a)**.

Use a method similar to that in the Example in Question 6.

$$2n - 1 = 45, \quad T_{23} - T_{21} = T_9$$

$$276 - 231 = 45$$

(9, 21, 23) [4]