Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **0580/42**

Paper 4 (Extended) **May/June 2020**

2 hours 30 minutes

You must answer on the question paper.

You will need: Geometrical instruments

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For π, use either your calculator value or 3.142.

INFORMATION

- The total mark for this paper is 130.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

1 (a) (i) Divide \$24 in the ratio 7 : 5.

$\frac{7}{12} \times 24$ $\frac{5}{12} \times 24$

\$ ....14.... , \$ ....10.... [2]

(ii) Write \$24.60 as a fraction of \$2870.
Give your answer in its lowest terms.

$\frac{24.60}{2870}$

....3/350.... [2]

(iii) Write \$1.92 as a percentage of \$1.60 .

$\frac{1.92}{1.60} \times 100$

....120.... % [1]

(b) In a sale the original prices are reduced by 15%.

(i) Calculate the sale price of a book that has an original price of \$12.

$12 = 100$
$? = 85$

$\frac{12 \times 85}{100}$

\$ ....10.20.... [2]

(ii) Calculate the original price of a jacket that has a sale price of \$38.25 .

$38.25 = 85\%$
$? = 100\%$

$\frac{38.25 \times 100}{85}$

\$ ....45.... [2]

(c) **(i)** Dean invests \$500 for 10 years at a rate of 1.7% per year simple interest.

Calculate the total interest earned during the 10 years.

$I = \frac{PRT}{100} = \frac{500 \times 1.7}{100} \times 10 = 85$

\$ ......85...... [2]

(ii) Ollie invests \$200 at a rate of 0.0035% **per day** compound interest.

Calculate the value of Ollie's investment at the end of 1 year.
[1 year = 365 days.]

$A = P\left(1 + \frac{r}{100}\right)^n$

$A = 200\left(1 + \frac{0.0035}{100}\right)^{365}$

$= 202.57$

\$ ......203...... [2]

(iii) Edna invests \$500 at a rate of r% per year compound interest.
At the end of 6 years, the value of Edna's investment is \$559.78 .

Find the value of r.

$559.78 = 500\left(1 + \frac{r}{100}\right)^6$

$\sqrt[6]{\frac{559.78}{500}} = 1 + \frac{r}{100}$

$1.019001 = 1 + \frac{r}{100}$

$1.019001 - 1 = \frac{r}{100}$

$r = 1.9$

r = ...1.9 %...... [3]

2 **(a)** $\mathbf{p} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

(i) Find $2\mathbf{p} + \mathbf{q}$.

$2\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 6 \\ 17 \end{pmatrix}$

$\begin{pmatrix} 6 \\ 17 \end{pmatrix}$ [2]

(ii) Find $|\mathbf{p}|$.

$\sqrt{4^2 + 5^2} = \sqrt{16 + 25}$ $\sqrt{41}$

6.4 .... [2]

(b) A is the point (4, 1) and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$.

Find the coordinates of B. (x, y)

$\begin{pmatrix} y \\ x \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

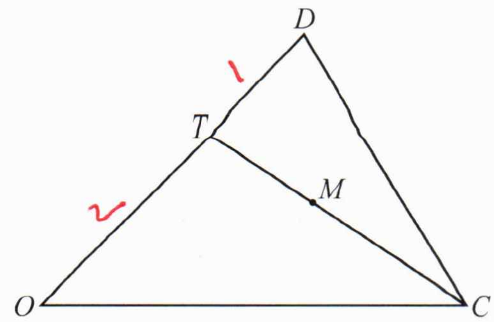
(......1......... , ...2.........) [1]

(c) The line $y = 3x - 2$ crosses the y-axis at G.

Write down the coordinates of G.

(......0......... , ...-2.........) [1]

(d)



NOT TO SCALE

In the diagram, O is the origin, $OT = 2TD$ and M is the midpoint of TC.
$\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$.

Find the position vector of M.
Give your answer in terms of $\mathbf{c}$ and $\mathbf{d}$ in its simplest form.

$\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$

$= \mathbf{c} + \frac{1}{2}\overrightarrow{CT}$

where $\overrightarrow{CT} = -\mathbf{c} + \frac{2}{3}\mathbf{d}$

$= \mathbf{c} + \frac{1}{2}\left(-\mathbf{c} + \frac{2}{3}\mathbf{d}\right)$

$= \mathbf{c} - \frac{1}{2}\mathbf{c} + \frac{1}{3}\mathbf{d}$

$\frac{1}{2}\mathbf{c} + \frac{1}{3}\mathbf{d}$ .... [3]

3 The speed, v km/h, of each of 200 cars passing a building is measured.
The table shows the results.

Mid point 10 30 42.5 47.5 55 70

Speed (v km/h)	$0 < v \leqslant 20$	$20 < v \leqslant 40$	$40 < v \leqslant 45$	$45 < v \leqslant 50$	$50 < v \leqslant 60$	$60 < v \leqslant 80$
Frequency	16	34	62	58	26	4

(a) Calculate an estimate of the mean.

use mid points

$$\frac{(10\times16)+(30\times34)+(42.5\times62)+(47.5\times58)+(55\times26)+(70\times4)}{200}$$

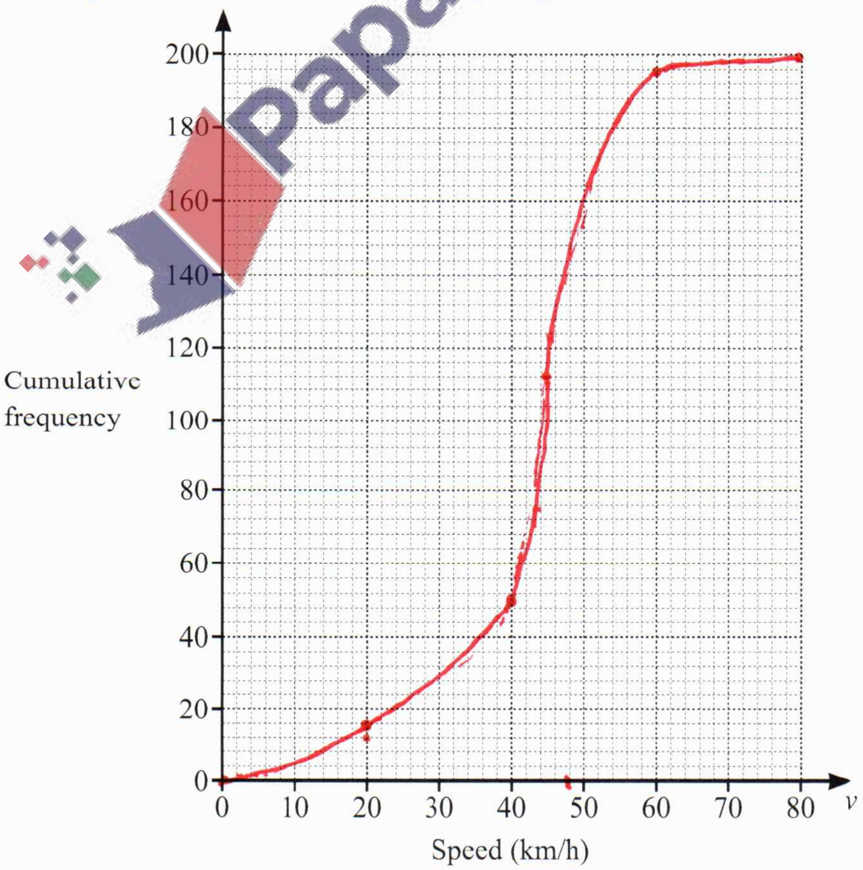
41.4 km/h [4]

(b) **(i)** Use the frequency table to complete the cumulative frequency table.

Speed (v km/h)	$v \leqslant 20$	$v \leqslant 40$	$v \leqslant 45$	$v \leqslant 50$	$v \leqslant 60$	$v \leqslant 80$
Cumulative frequency	16	50	112	170	196	200

[1]

(ii) On the grid, draw a cumulative frequency diagram.



[3]

(iii) Use your diagram to find an estimate of

(a) the upper quartile,

$\frac{3}{4} \times 200$

......48...... km/h [1]

(b) the number of cars with a speed greater than 35 km/h.

$200 - 40$

......160...... [2]

(c) Two of the 200 cars are chosen at random.

Find the probability that they both have a speed greater than 50 km/h.

without replacement $\frac{30}{200} \times \frac{29}{199}$

......$87/3980$...... [2]

(d) A new frequency table is made by combining intervals.

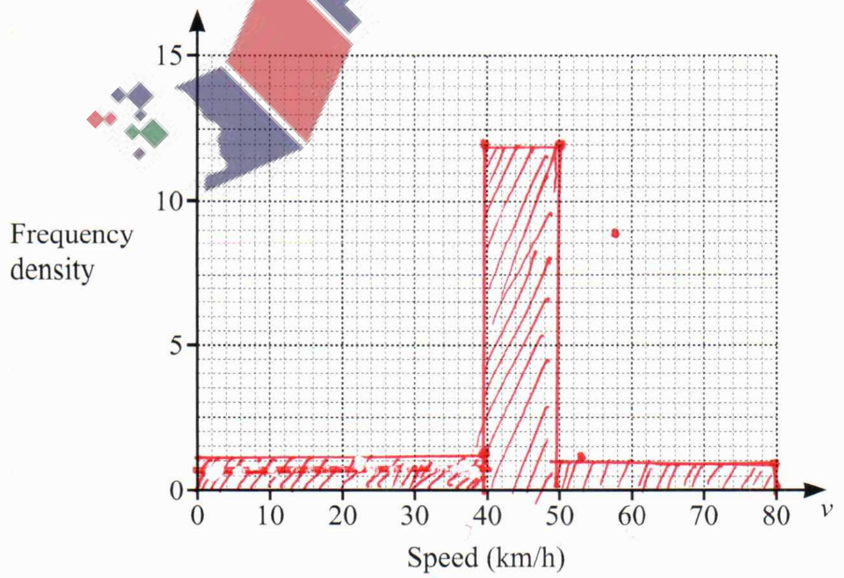
Speed (v km/h)	$0 < v \leqslant 40$	$40 < v \leqslant 50$	$50 < v \leqslant 80$
Frequency	50	120	30
F.d	1.25	12	

$F.d = \frac{f}{c.w}$

$\frac{50}{40}$

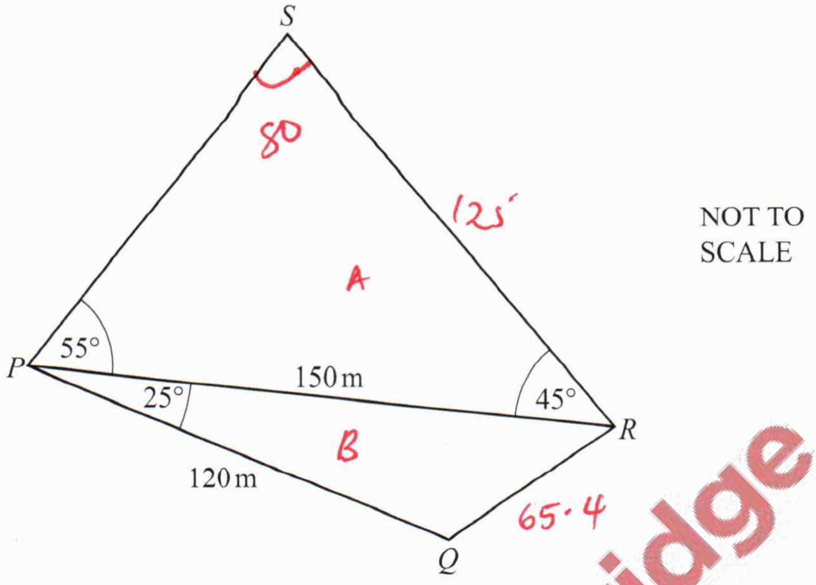
$\frac{30}{30}$

On the grid, draw a histogram to show the information in this table.



[3]

4



The diagram shows two triangles.

(a) Calculate QR.

use cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 150^2 + 120^2 - 2 \times 150 \times 120 \cos 25$

$x^2 = 4,272.9....$

$x = \sqrt{4,272.9}$

$QR =$ ....65.4.... m [3]

(b) Calculate RS.

use sine rule

$PSR = 80°$

$180 - (55 + 45)$

$\frac{150}{\sin 80} = \frac{RS}{\sin 55}$

$RS = \frac{150 \sin 55}{\sin 80}$

$= 124.768$

$RS =$ ....125.... m [4]

(c) Calculate the total area of the two triangles.

Area A = $\frac{1}{2}ab\sin C$ = $\frac{1}{2} \times 150 \times 125 \sin 45$

= 6,629.13

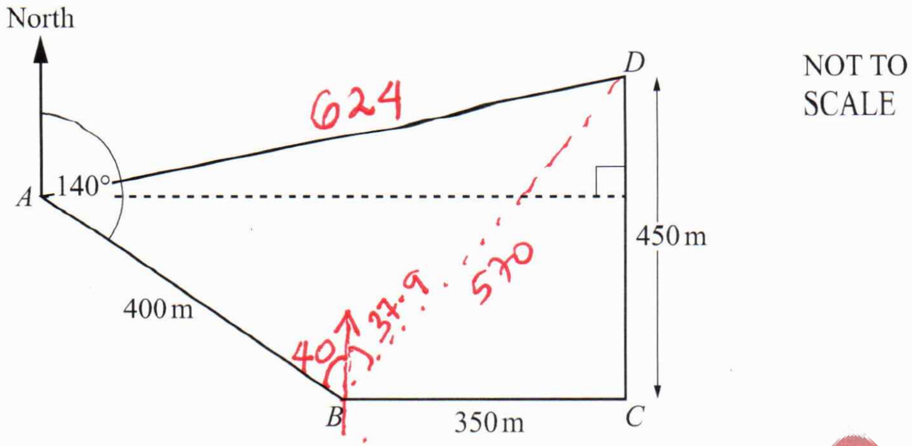
B = $\frac{1}{2} \times 150 \times 120 \sin 25$

= 3,803.56

= 10432.69

10,433. .... m^2 [3]

5



The diagram shows a field *ABCD*.
The bearing of *B* from *A* is 140°.
C is due east of *B* and *D* is due north of *C*.
AB = 400 m, *BC* = 350 m and *CD* = 450 m.

(a) Find the bearing of *D* from *B*.

Angle DBC

$\tan x = \frac{450}{350}$

$x = \tan^{-1}\left(\frac{450}{350}\right) = 52.1$

$90 - 52.1 = 37.9 \simeq 38$

.... 038 .... [2]

(b) Calculate the distance from D to A.

use Pythagoras

$BD = \sqrt{(450^2 + 350^2)}$

$BD = \sqrt{325,000} = 570.$

Using cosine rule to find AD

$AD^2 = 400^2 + 570^2 - 2 \times 400 \times 570 \cos 77.9$

$AD^2 = 389,313.9$

$AD = 623.95$

624 .... m [6]

(c) Jono runs around the field from A to B, B to C, C to D and D to A.
He runs at a speed of 3 m/s.

Calculate the total time Jono takes to run around the field.
Give your answer in minutes and seconds, correct to the nearest second.

Perimeter $= 624 + 400 + 350 + 450 = 1,824$

$t = D/s = \frac{1824}{3} = 608$

....10.... min ....8.... s [4]

  [Turn over

6 $\quad f(x) = 3x + 2 \qquad g(x) = x^2 + 1 \qquad h(x) = 4^x$

(a) Find h(4).

$h(4) = 4^4$

256 [1]

(b) Find fg(1).

$g(1) = x^2 + 1 = 1^2 + 1 = 2$

$f(2) = 3 \times 2 + 2$

8 [2]

(c) Find gf(x) in the form $ax^2 + bx + c$.

$(3x+2)^2 + 1$

$\downarrow$

$9x^2 + 12x + 4 + 1$

$(3x+2)^2 = (3x+2)(3x+2)$

$9x^2 + 6x + 6x + 4$

$9x^2 + 12x + 5$ [3]

(d) Find x when $f(x) = g(7)$.

$g(x) = 7^2 + 1 = 50$

$3x + 2 = 50$

$\frac{3x}{3} = \frac{48}{3}$

$x =$ 16 [2]

(e) Find $f^{-1}(x)$.

$3x + 2 = y$

$3x = y - 2$

$x = \frac{y-2}{3}$

$f^{-1}(x) = \frac{x-2}{3}$ [2]

(f) Find $\dfrac{g(x)}{f(x)} + x$.

Give your answer as a single fraction, in terms of x, in its simplest form.

$$\frac{x^2+1}{3x+2} + \frac{x}{1}$$

$$\frac{x^2+1+x(3x+2)}{3x+2}$$

$$x^2+1+3x^2+2x$$

$$\frac{4x^2+2x+1}{3x+2}$$

$\dfrac{4x^2+2x+1}{3x+2}$ [3]

(g) Find x when $h^{-1}(x) = 2$.

4^2

$x =$ 16 [1]

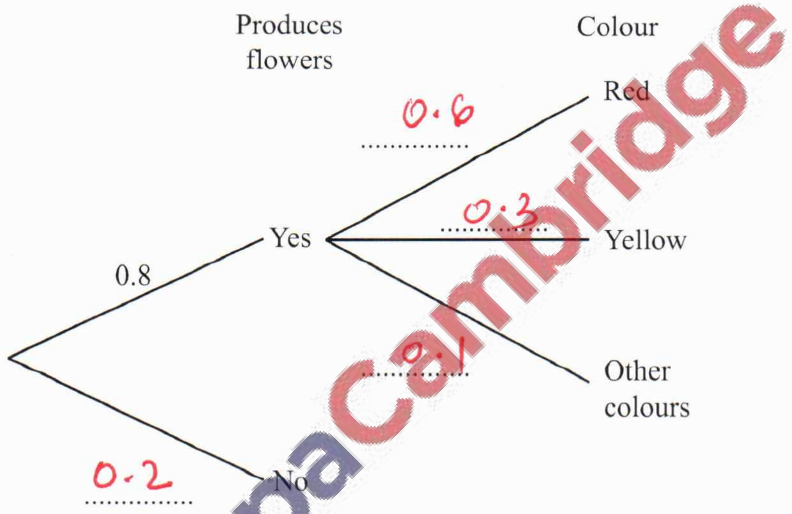
7 Tanya plants some seeds.
The probability that a seed will produce flowers is 0.8 .
When a seed produces flowers, the probability that the flowers are red is 0.6 and the probability that the flowers are yellow is 0.3 .

(a) Tanya has a seed that produces flowers.

Find the probability that the flowers are not red and not yellow.

.... [1]

(b) **(i)** Complete the tree diagram.



[2]

(ii) Find the probability that a seed chosen at random produces red flowers.

0.8×0.6

0.48 .... [2]

(iii) Tanya chooses a seed at random.

Find the probability that this seed does not produce red flowers and does not produce yellow flowers.

P(No flower) + P(flower & others)

0.2 + 0.8 × 0.1

0.2 + 0.08

0.28. .... [3]

(c) Two of the seeds are chosen at random.

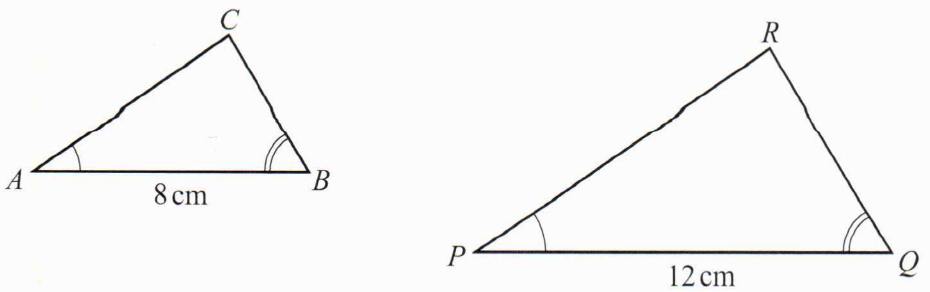
Find the probability that one produces flowers and one does not produce flowers.

(0.8 × 0.2) or (0.2 × 0.8)

0.16 + 0.16

0.32 .... [3]

8 **(a)**



Triangle ABC is mathematically similar to triangle PQR.
The area of triangle ABC is $16\,\text{cm}^2$.

(i) Calculate the area of triangle PQR.

$l.s.f = \frac{12}{8} = 1.5$

$A.s.f = (l.s.f)^2 = 1.5^2 \times 16$

.......36....... cm^2 [2]

(ii) The triangles are the cross-sections of prisms which are also mathematically similar.
The volume of the smaller prism is $320\,\text{cm}^3$.

Calculate the length of the larger prism.

$l.s.f = 1.5$

$V.s.f = 1.5^3 = 3.375$

volume of the larger one $320 \times 3.375 = 1080$

length of the prism.

$= \frac{1080}{36}$

.......30....... cm [3]

Vol of a prism = X-sectional area x length

(b) A cylinder with radius 6 cm and height h cm has the same volume as a sphere with radius 4.5 cm.

Find the value of h.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Vol. of a cylinder $= \pi r^2 h = \pi \times 6^2 \times h = \frac{4}{3} \times \pi \times 4.5^3$

$h = \dfrac{\frac{4}{3} \times \cancel{\pi} \times 4.5^3}{\cancel{\pi} \times 6^2} = 3.375$

$h =$ ......3.375...... [3]

(c) A solid metal cube of side 20 cm is melted down and made into 40 solid spheres, each of radius r cm.

Find the value of r.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Volume of the cube $= 20 \times 20 \times 20 = 8000\ cm^3$

Vol. of one sphere $= \frac{8000}{40} = 200$

$\frac{4}{3}\pi r^3 = 200$

$r^3 = \dfrac{200}{\frac{4}{3}\pi}$

$r^3 = 47.7$

$r =$ ......3.63...... [3]

(d) A solid cylinder has radius x cm and height $\frac{7x}{2}$ cm.

The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder.

Find an expression for R in terms of x.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

S.A of a cylinder $2\pi r^2 + 2\pi r h$.

$\dfrac{2 \times \pi \times x^2}{2\pi} + \dfrac{2\pi \times x \times \frac{7x}{2}}{2\pi} = \dfrac{4\pi R^2}{2\pi}$

$x^2 + \dfrac{7x^2}{2} = 2R^2$

$R^2 = \frac{9}{4}x^2$

$R = \sqrt{\frac{9}{4}x^2}$

$R =$ ......$\frac{3}{2}x$...... [3]

9 **(a)** **(i)** Write x^2+8x-9 in the form $(x+k)^2+h$.

given x^2+bx+c

$\left(x+\frac{b}{2}\right)^2+c-\left(\frac{b}{2}\right)^2$

$\left(x+\frac{8}{2}\right)^2-9-\left(\frac{8}{2}\right)^2$

$(x+4)^2-25$ .... [2]

(ii) Use your answer to **part (a)(i)** to solve the equation $x^2+8x-9=0$.

$(x+4)^2=25$

$(x+4)=\pm\sqrt{25}$

$x+4=5$

$x+4=-5$

$x =$ ......1...... or $x =$ ......-9...... [2]

(b) The solutions of the equation $x^2+bx+c=0$ are $\frac{-7+\sqrt{61}}{2}$ and $\frac{-7-\sqrt{61}}{2}$.

Find the value of b and the value of c.

$b=7$ then $b^2-4ac=61$

$7^2-4(1)(c)=61$

$-4c=61-49$

$-4c=12$

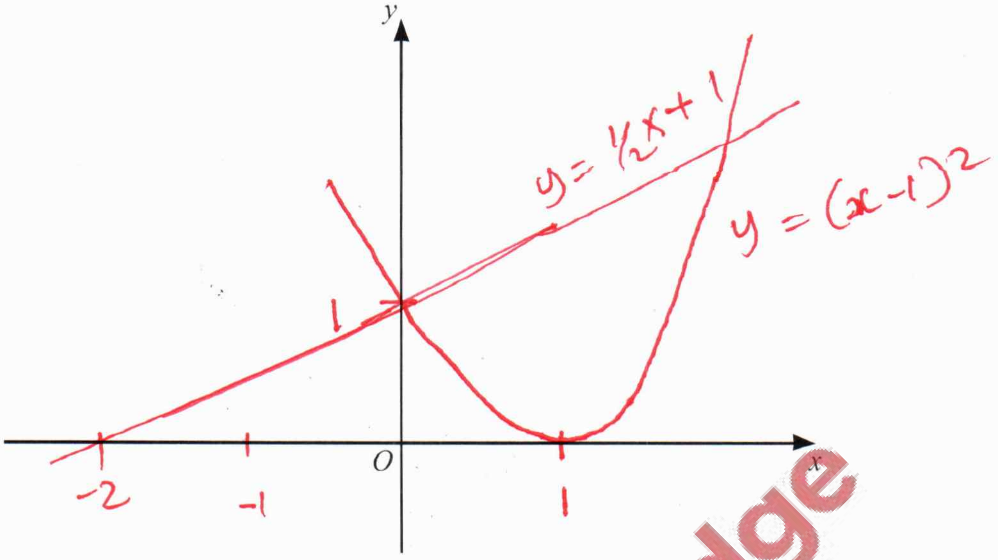
$c=12/-4$

$c=-3$

$b =$ ......7......

$c =$ ......-3...... [3]

(c) **(i)**



On the diagram,

(a) sketch the graph of $y = (x-1)^2$, [2]

(b) sketch the graph of $y = \frac{1}{2}x + 1$. [2]

(ii) The graphs of $y = (x-1)^2$ and $y = \frac{1}{2}x + 1$ intersect at A and B.

Find the length of AB.

$|AB| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$(0, 1)$ and $(5/2, 2.25)$

$\sqrt{(2.25 - 1)^2 + (2.5 - 0)^2}$

$= \sqrt{7.8125}$

2.795

$\frac{1}{2}x + 1 = (x-1)^2$

$\frac{1}{2}x + 1 = x^2 - 2x + 1$

$x + \cancel{2} = 2x^2 - 4x + \cancel{2}$

$2x^2 - 5x$

$x(2x - 5) = 0$

$x = 0 \quad x = 5/2$

$y = 1 \quad y = \frac{1}{2} \times 5/2 + 1$

$y = 2.25$

$AB = $ .... 2.8 .... [7]

Question 10 is printed on the next page.

10 **(a)** $y = x^4 - 4x^3$

(i) Find the value of y when $x = -1$.

$y = (-1)^4 - 4(-1)^3$

$y =$ ....5....... [2]

(ii) Find the two stationary points on the graph of $y = x^4 - 4x^3$.

$\frac{dy}{dx} = 4x^3 - 12x^2$ at stationary point $\frac{dy}{dx} = 0$

$4x^2(x-3) = 0$

$4x^2 = 0$ $x - 3 = 0$

$x = 0$ $x = 3$

$y = 0$ $y\ 3^4 - 4(3)^3$

(.......3....... , ...−27.......)

(.......0....... , ...0.......) [6]

(b) $y = x^p + 2x^q$

$\frac{dy}{dx} = 11x^{10} + 10x^4$, where $\frac{dy}{dx}$ is the derived function.

Find the value of p and the value of q.

$p = 11$ $q = 5$

$p =$ ....11.......

$q =$ ....5....... [2]