UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

9523289317

MATHEMATICS **0580/21**

Paper 2 (Extended) **May/June 2011**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator, Geometrical instruments, Mathematical tables (optional), Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 70.

This document consists of **12** printed pages.

IB11 06_0580_21/7RP

UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

For Examiner's Use

1 A concert hall has 1540 seats.

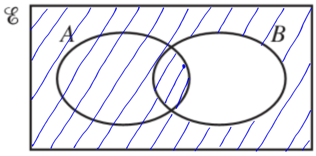
Calculate the number of people in the hall when 55% of the seats are occupied.

$$N = \frac{55}{100} \times 1540$$

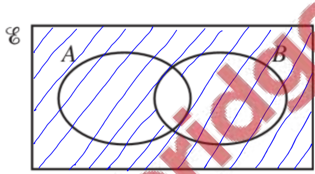
$$\Rightarrow N = 847$$

Answer ....847........ [1]

2 Shade the required region on each Venn diagram.



$A \cup B'$



$(A \cap B)'$

[2]

3 Calculate $81^{0.25} \div 4^{-2}$.

Answer ....48........ [2]

4 **(a)** Find m when $4^m \times 4^2 = 4^{12}$.

• Since the bases are equal,

$\Rightarrow m + 2 = 12$

$\Rightarrow m = 10$

Answer(a) m = ...10........ [1]

(b) Find p when $6^p \div 6^5 = \sqrt{6}$.

$6^p \div 6^5 = 6^{1/2}$

• Since the bases are equal,

$\Rightarrow p - 5 = 1/2$

$\Rightarrow p = 5.5$

Answer(b) p = ...5.5........ [1]

For Examiner's Use

5 A hummingbird beats its wings 24 times per second.

(a) Calculate the number of times the hummingbird beats its wings in one hour.

$\star\ N = \frac{24 \text{ times}}{\text{sec}} \times 1\text{hr}$

$\Rightarrow N = \frac{24 \text{ times}}{\text{sec}} \times 3600\,\cancel{\text{sec}}$

$\Rightarrow N = \underline{86\,400 \text{ times}}$

Answer(a) ....86 400....... [1]

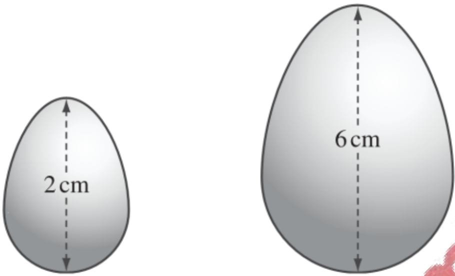
(b) Write your answer to **part (a)** in standard form.

8.6 4 0 0 (4 3 2 1)

$\underline{8\cdot 64 \times 10^4}$

Answer(b) ....$8\cdot 64 \times 10^4$....... [1]

6



NOT TO SCALE

A company makes solid chocolate eggs and their shapes are mathematically similar.
The diagram shows eggs of height 2 cm and 6 cm.
The mass of the small egg is 4 g.

Calculate the mass of the large egg.

Note: Density is proportional to Volume, hence we'll use the Volume scale factor.

$(2)^3 : (6)^3$

$4g : x$

$\Rightarrow x = \frac{6^3}{2^3} \times 4g$

$\Rightarrow x = \underline{108\,g}$

Answer ....108....... g [2]

7 Find the length of the straight line from Q (–8, 1) to R (4 , 6).

$\star\ L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow L = \sqrt{(-8-4)^2 + (1-6)^2}$

$\Rightarrow L = \underline{13}$

Answer QR = ...13....... [3]

8 Calculate the radius of a sphere with volume 1260 cm^3.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$V = \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = \frac{3V}{4\pi}$

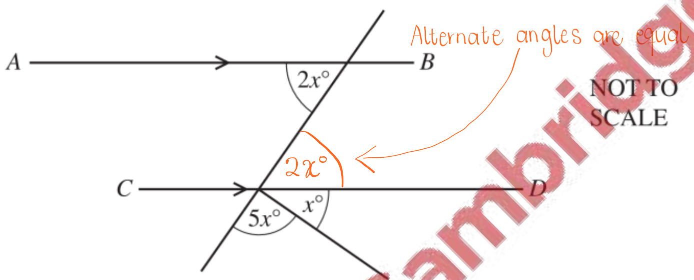
$\Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$

$\Rightarrow r = \left(\sqrt[3]{\frac{3 \times 1260}{4\pi}}\right)$ cm

$\Rightarrow r = \underline{6.70}$ cm (3 sig. figs.)

Answer ......6.70...... cm [3]

9



AB is parallel to *CD*.
Calculate the value of x.

• Angles on a straight line add up to 180°

$\Rightarrow 5x + x + 2x = 180°$

$\Rightarrow 8x = 180°$

$\Rightarrow x = \underline{22.5°}$

Answer $x =$ ......22.5...... [3]

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10 Solve the simultaneous equations.

$3x + y = 30$ — (1)
$2x - 3y = 53$ — (2)

(1) × 3 : $9x + 3y = 90$ — (3)

(3) + (2) : $(9+2)x = 90 + 53$

$\Rightarrow 11x = 143$

$\Rightarrow \underline{x = 13}$

• Finding y :

$\Rightarrow 3(13) + y = 30$

$\Rightarrow y = \underline{-9}$

Answer $x =$ ......13......

$y =$ ......−9...... [3]

11 A rectangular photograph measures 23.3 cm by 19.7 cm, each correct to 1 decimal place.
Calculate the lower bound for

(a) the perimeter,

☆ $L = \left(23.3 \pm \frac{0.1}{2}\right)$cm

☆ $W = \left(19.7 \pm \frac{0.1}{2}\right)$cm

☆ $LB(P) = (2 \times LB(L)) + (2 \times LB(W))$

$\Rightarrow LB(P) = (2 \times 23.25)$cm $+ (2 \times 19.65)$cm

$\Rightarrow LB(P) = 85.8$ cm

Answer(a) ....85.8........ cm [2]

(b) the area.

☆ $LB(A) = LB(L) \times LB(W)$

$\Rightarrow LB(A) = (23.25 \times 19.65)\text{cm}^2$

$\Rightarrow LB(A) = 456.8625\text{cm}^2$

Answer(b) ....456.8625........ cm^2 [1]

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12 A train leaves Barcelona at 21 28 and takes 10 hours and 33 minutes to reach Paris.

(a) Calculate the time the next day when the train arrives in Paris.

hrs.	mins.
~~24~~ 23	00 +60
− 21	28
2	32

hrs.	mins.
10	33
− 2	32
08	01

Answer(a) ....08 01........ [1]

(b) The distance from Barcelona to Paris is 827 km.

Calculate the average speed of the train in kilometres per hour.

☆ Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$

$\Rightarrow$ Average speed = $\frac{827\text{ km}}{10\frac{33}{60}\text{ h}}$

$\Rightarrow$ Average speed = 78.4 km/h (3 sig. figs.)

Answer(b) ....78.4........ km/h [3]

For Examiner's Use

13 The scale on a map is 1: 20 000.

(a) Calculate the actual distance between two points which are 2.7 cm apart on the map.
Give your answer in kilometres.

$1 : 20\,000$

$2.7\text{cm} : x$

$\Rightarrow x = \frac{20\,000}{1} \times 2.7\text{cm}$

$\Rightarrow x = 20\,000 \times 2.7 \times 10^{-5}\text{km}$

$\Rightarrow x = 0.54\text{km}$

Answer(a) 0.54 km [2]

(b) A field has an area of $64\,400\,\text{m}^2$.
Calculate the area of the field on the map in cm^2.

$(1)^2 : (20\,000)^2$

$x : 64\,400\text{m}^2$

$\Rightarrow x = \frac{(1)^2}{(20\,000)^2} \times 64\,400\text{m}^2$

$\Rightarrow x = \frac{(1)^2}{(20\,000)^2} \times 64\,400 \times 10^4\text{cm}^2$

$\Rightarrow x = 1.61\text{cm}^2$

Answer(b) 1.61 cm^2 [2]

14 Solve the equation $2x^2 + 3x - 6 = 0$.
Show all your working and give your answers correct to 2 decimal places.

$\star\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

• $a = 2$, $b = 3$, $c = -6$

$\Rightarrow x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-6)}}{2(2)}$

$\Rightarrow x = \frac{-3 \pm \sqrt{57}}{4}$

$x_1 = \frac{-3 + \sqrt{57}}{4} \simeq 1.14$ (2dp)

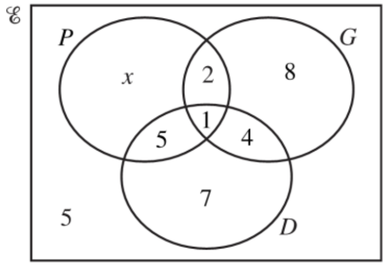
$x_2 = \frac{-3 - \sqrt{57}}{4} \simeq -2.64$ (2dp)

Answer x = 1.14 or *x* = −2.64 [4]

15 A teacher asks 36 students which musical instruments they play.

P = {students who play the piano}
G = {students who play the guitar}
D = {students who play the drums}

The Venn diagram shows the results.



(a) Find the value of x.

$\star\ x + 2 + 8 + 5 + 1 + 4 + 7 + 5 = 36$

$\Rightarrow x + 32 = 36$

$\Rightarrow x = 4$

Answer(a) x = 4 [1]

(b) A student is chosen at random.

Find the probability that this student

(i) plays the drums but **not** the guitar,

$\star\ P = \dfrac{n(D \cap G')}{n(\mathscr{E})}$

$\Rightarrow P = \dfrac{7+5}{36} = \dfrac{12}{36} = \dfrac{1}{3}$

Answer(b)(i) $\dfrac{1}{3}$ [1]

(ii) plays only 2 different instruments.

$\star\ P = \dfrac{2+5+4}{36} = \dfrac{11}{36}$

Answer(b)(ii) $\dfrac{11}{36}$ [1]

(c) A student is chosen at random from those who play the guitar.

Find the probability that this student plays no other instrument.

$\star\ P = \dfrac{n(G\text{ only})}{n(G)}$

$\Rightarrow P = \dfrac{8}{8+2+1+4} = \dfrac{8}{15}$

Answer(c) $\dfrac{8}{15}$ [1]

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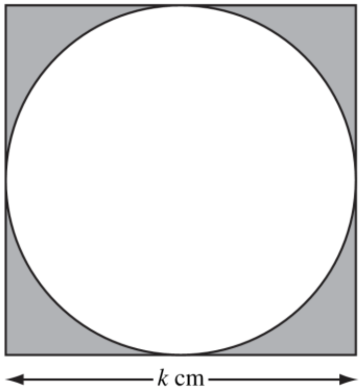

The diagram shows a square of side k cm.

The circle inside the square touches all four sides of the square.

(a) The shaded area is $A\,\text{cm}^2$.

Show that $\quad 4A = 4k^2 - \pi k^2$.

Answer (a)

$A_{SR} = A_{SQUARE} - A_{CIRCLE}$

$\Rightarrow A = (k \times k) - \pi\left(\frac{k}{2}\right)^2$

$\Rightarrow A = k^2 - \frac{\pi k^2}{4}$

Multiply through by 4:

$\Rightarrow 4A = 4k^2 - \pi k^2$

[2]

(b) Make k the subject of the formula $\quad 4A = 4k^2 - \pi k^2$.

$4A = 4k^2 - \pi k^2$

$\Rightarrow 4A = k^2(4 - \pi)$

$\Rightarrow k^2 = \frac{4A}{4 - \pi}$

$\Rightarrow k = \pm\sqrt{\frac{4A}{4-\pi}}$

Answer(b) $k =$ $\pm\sqrt{\frac{4A}{4-\pi}}$ [3]

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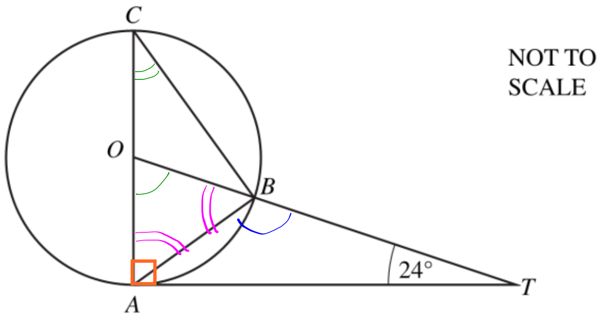

A, B and C are points on a circle, centre O.
TA is a tangent to the circle at A and OBT is a straight line.
AC is a diameter and angle $OTA = 24°$.

Calculate

(a) angle AOT,

$\star \ A\hat{O}T + 90° + 24° = 180°$

$\Rightarrow A\hat{O}T + 114° = 180°$

$\Rightarrow A\hat{O}T = \underline{66°}$

Answer(a) Angle AOT = ...66°........ [2]

(b) angle ACB,

$\star \ A\hat{C}B = \frac{1}{2} \times A\hat{O}B$

$\Rightarrow A\hat{C}B = \frac{1}{2} \times 66° = \underline{33°}$

Answer(b) Angle ACB = ...33°........ [1]

(c) angle ABT.

$\star \ (2 \times A\hat{B}O) + 66° = 180°$

$\Rightarrow 2 \times A\hat{B}O = 114°$

$\Rightarrow A\hat{B}O = \underline{57°}$

$\star \ A\hat{B}T + A\hat{B}O = 180°$

$\Rightarrow A\hat{B}T = 180° - 57° = \underline{123°}$

Answer(c) Angle ABT = ...123°........ [2]

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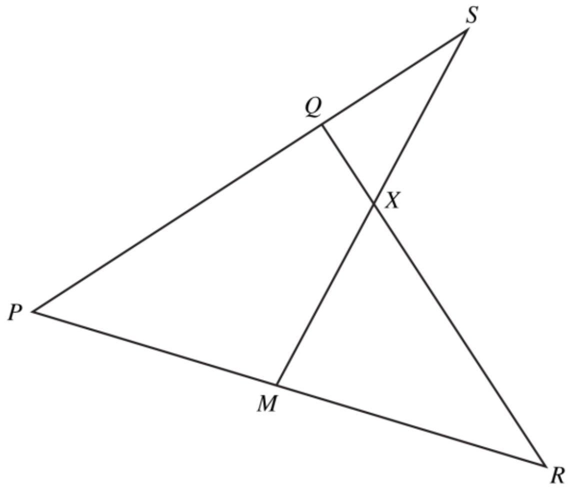

In the diagram, *PQS*, *PMR*, *MXS* and *QXR* are straight lines.

$PQ = 2\,QS$.
M is the midpoint of *PR*.
$QX : XR = 1 : 3$.

$\overrightarrow{PQ} = \mathbf{q}$ and $\overrightarrow{PR} = \mathbf{r}$.

(a) Find, in terms of **q** and **r**,

(i) $\overrightarrow{RQ}$,

$\star\ \overrightarrow{RQ} = \overrightarrow{RP} + \overrightarrow{PQ}$

$\Rightarrow \overrightarrow{RQ} = -r + q$

Answer(a)(i) $\overrightarrow{RQ}$ = $-r + q$ [1]

(ii) $\overrightarrow{MS}$.

$\star\ \overrightarrow{MS} = \overrightarrow{MP} + \overrightarrow{PS}$

$\Rightarrow \overrightarrow{MS} = \overrightarrow{MP} + \overrightarrow{PQ} + \overrightarrow{QS}$

$\Rightarrow \overrightarrow{MS} = -\frac{1}{2}r + q + \frac{1}{2}q$

$\Rightarrow \overrightarrow{MS} = -\frac{1}{2}r + \frac{3}{2}q$

Answer(a)(ii) $\overrightarrow{MS}$ = $-\frac{1}{2}r + \frac{3}{2}q$ [1]

(b) By finding $\overrightarrow{MX}$, show that *X* is the midpoint of *MS*.

Answer (b)

$\star\ \overrightarrow{MX} = \overrightarrow{MR} + \overrightarrow{RX}$

$\Rightarrow \overrightarrow{MX} = \overrightarrow{MR} + \frac{3}{4}\overrightarrow{RQ}$

$\Rightarrow \overrightarrow{MX} = \frac{1}{2}r + \frac{3}{4}(-r + q)$

$\Rightarrow \overrightarrow{MX} = -\frac{1}{4}r + \frac{3}{4}q$

If X is the midpoint of MS, then MX must be equal to 1/2 of MS.

$\star\ \frac{1}{2}\overrightarrow{MS} = \frac{1}{2}\left(-\frac{1}{2}r + \frac{3}{2}q\right)$

$\Rightarrow \frac{1}{2}\overrightarrow{MS} = -\frac{1}{4}r + \frac{3}{4}q = \overrightarrow{MX}$

[3]

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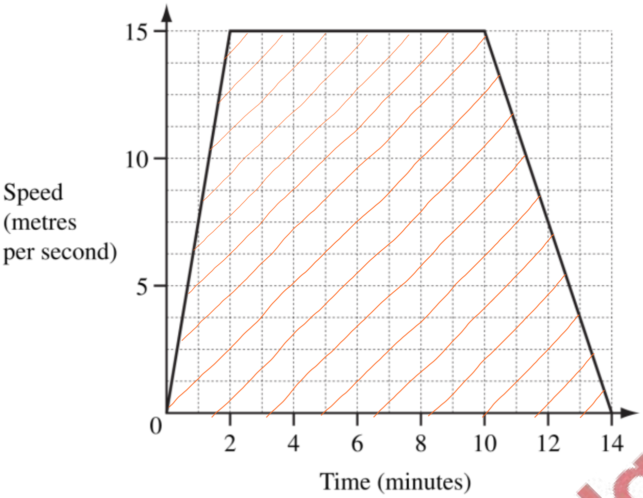

The diagram shows the speed-time graph of a train journey between two stations.
The train accelerates for two minutes, travels at a constant maximum speed, then slows to a stop.

(a) Write down the number of **seconds** that the train travels at its constant maximum speed.

$\Rightarrow T = (10-2)$ min

$\Rightarrow T = 8 \times 60$s

$\Rightarrow T = 480$s

Answer(a) ....480......... s [1]

(b) Calculate the distance between the two stations **in metres**.

Total distance (D) = Area under the graph

$\Rightarrow D = \frac{1}{2}(a+b) \times h$

$\Rightarrow D = \frac{1}{2}(8+4)$ mins $\times 15\frac{m}{s}$

$\Rightarrow D = \frac{1}{2}(8+4) \times 60\cancel{s} \times 15\frac{m}{\cancel{s}}$

$\Rightarrow D = 9900$m

Answer(b) ....9900......... m [3]

(c) Find the acceleration of the train in the **first two minutes**.
Give your answer in m/s².

$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$

$\Rightarrow a = \frac{(15-0)\text{ m/s}}{(2-0)\text{ mins}} = \frac{(15-0)\text{ m/s}}{(2-0) \times 60\text{s}}$

$\Rightarrow a = 0.125\text{ m/s}^2$

Answer(c) ....0.125......... m/s² [2]

Question 20 is printed on the next page.

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20 $\quad f(x) = x^3 \qquad g(x) = 2x - 3$

(a) Find

(i) g(6),

$\star\ g(6) = 2(6) - 3$

$\Rightarrow g(6) = 9$

Answer(a)(i) ......9...... [1]

(ii) f(2*x*).

$\star\ f(2x) = (2x)^3$

$\Rightarrow f(2x) = 2^3 x^3$

$\Rightarrow f(2x) = 8x^3$

Answer(a)(ii) ......$8x^3$...... [1]

(b) Solve fg(*x*) = 125.

$\star\ fg(x) = 125$

$\Rightarrow (2x-3)^3 = 125$

$\Rightarrow 2x - 3 = \sqrt[3]{125}$

$\Rightarrow 2x - 3 = 5$

$\Rightarrow x = \frac{5+3}{2}$

$\Rightarrow x = 4$

Answer(b) x = ...4...... [3]

(c) Find the inverse function $g^{-1}(x)$.

$\star\ g(x) = 2x - 3$

$\Rightarrow x = 2y - 3$

$\Rightarrow 2y = x + 3$

$\Rightarrow y = \frac{x+3}{2}$

$\therefore g^{-1}(x) = \frac{x+3}{2}$

Answer(c) $g^{-1}(x)$ = ...$\frac{x+3}{2}$...... [2]