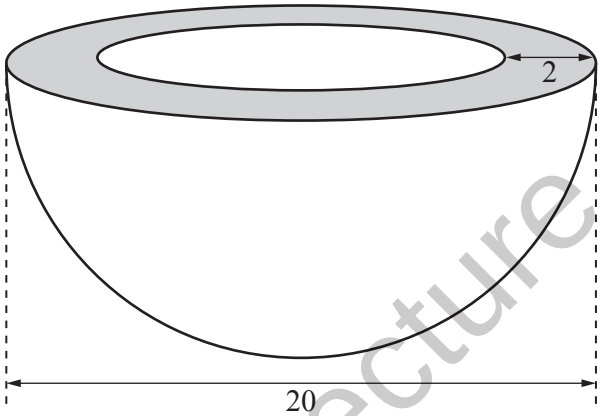
Name:

Section:

Mensuration Worksheet

1 **[Volume of a sphere $= \frac{4}{3}\pi r^3$]**

[Surface area of a sphere $= 4\pi r^2$]



The diagram shows a wooden bowl.
It is made in the shape of a large hemisphere with a small hemisphere removed from the centre.
The diameter of the large hemisphere is 20 cm.
The width of the rim of the bowl is 2 cm.

(a) Show that the total surface area of the bowl is $364\pi\,\text{cm}^2$.

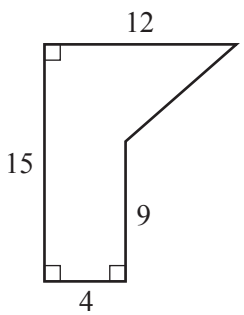
[3]

2 The perimeter of a regular hexagon is equal to the perimeter of a regular octagon.
Each edge of the octagon is 9 cm long.

Find the length of one edge of the hexagon.

..... cm [2]

3 **(a)**



NOT TO SCALE

The diagram shows a pentagon.
All the lengths are in centimetres.

(i) Calculate the area of the pentagon.

..... cm^2 [2]

(ii) Find the perimeter of the pentagon.

..... cm [3]

(b) **[Volume of a sphere $= \frac{4}{3}\pi r^3$]**

A sphere has a volume of $2572\,\text{cm}^3$.

Find the radius of the sphere.

..... cm [3]

(c)

A cuboid has dimensions 2 cm by 6 cm by 22.5 cm.

(i) Calculate the surface area of the cuboid.

..... cm^2 [3]

(ii) A cube of edge x cm has the same surface area as the cuboid.

Form an equation in x and solve it to find the length of the edge of the cube.
Show your working.

..... cm [3]

4

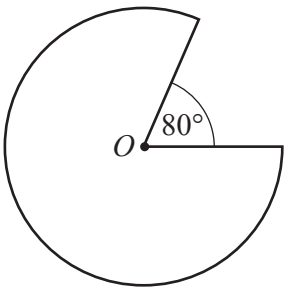
NOT TO SCALE

The area of the rectangle is 9 cm^2.
The area of the triangle is 85 mm^2.

Calculate the shaded area.
Give your answer in cm^2.

..... cm^2 [2]

5



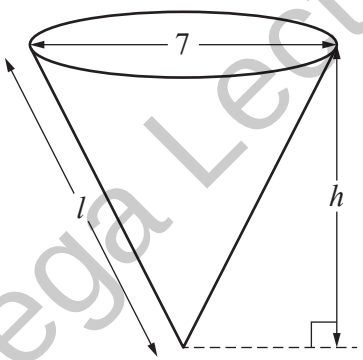
NOT TO SCALE

The diagram shows the major sector of a circle with centre O and radius 3 cm.

Calculate the area of this sector.
Give your answer in the form $k\pi$, where k is an integer.

..... cm^2 [2]

6



[Volume of cone $= \frac{1}{3}\pi r^2 h$]

[Curved surface area of a cone $= \pi r l$]

The diagram shows a paper cup in the shape of a cone.
The diameter of the top of the cup is 7 cm.
The volume of the cup is 110 cm^3.

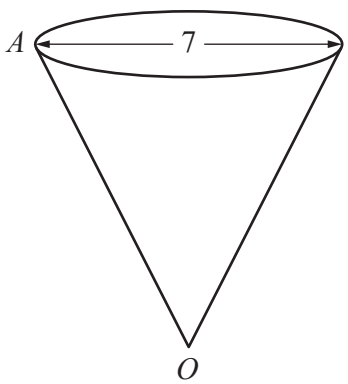
(a) Show that the height of the cup, h cm, is 8.57 correct to 2 decimal places.

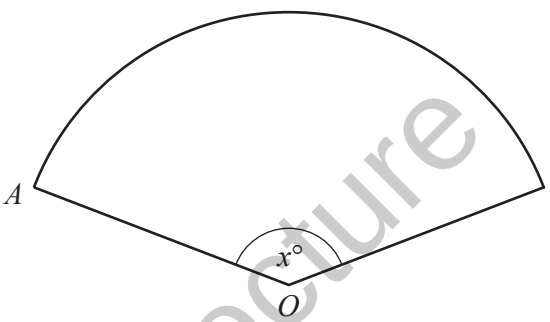
[3]

(b) Calculate the slant height, l cm, of the cup.

$l =$ ..... [2]

(c)





NOT TO SCALE

The cup is cut along the line OA.
It is opened out into a sector of a circle with centre O and sector angle $x°$.

Calculate the value of x.

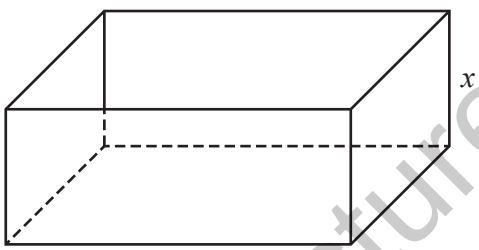
$x =$ ..... [4]

(d) A second paper cup is mathematically similar to the cup with volume $110\,\text{cm}^3$.
The volume of the second cup is $165\,\text{cm}^3$.

Calculate the diameter of the top of the second cup.

..... cm [2]

7



The diagram shows an open box in the shape of a cuboid.

The height of the box is x cm.
The width of the box is 5 cm more than its height.
The length of the box is two times its width.

(a) Write down expressions, in terms of x, for the width and the length of the box.

= ..... cm

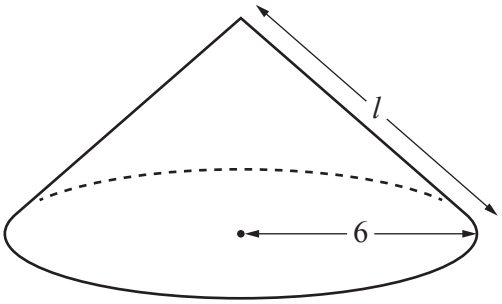
= ..... cm [2]

(b) The external surface area of the open box is $210\,\text{cm}^2$.

Form an equation in x and show that it simplifies to $4x^2 + 25x - 80 = 0$.

8 **[Volume of a cone = $\frac{1}{3}\pi r^2 h$]**

[Curved surface area of a cone = πrl]



A cone has radius 6 cm and slant height l cm.
The **total** surface area of the cone is 84π cm^2.

(a) Show that $l = 8$.

[2]

(b) Calculate the volume of the cone.

..... cm^3 [3]

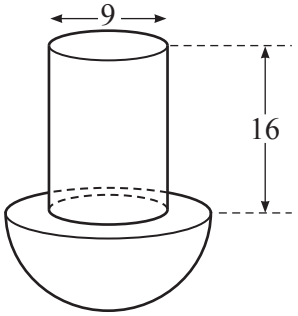
(c) A similar cone has a **total** surface area of 47.25π cm^2.

Find the radius of this cone.

..... cm [2]

9 (a) **[Volume of a sphere $= \frac{4}{3}\pi r^3$]**

[Surface area of a sphere $= 4\pi r^2$]



The diagram shows a solid formed by joining a cylinder to a hemisphere.
The diameter of the cylinder is 9 cm and its height is 16 cm.

(i) The volume of the hemisphere is equal to the volume of the cylinder.

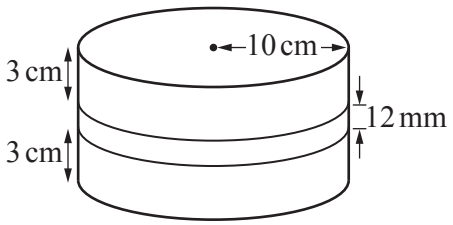
Show that the radius of the hemisphere is 7.86 cm, correct to 2 decimal places.

[4]

(ii) Calculate the total surface area of the solid.

..... cm^2 [3]

10 A birthday cake is in the shape of a cylinder.
There are two layers of cake and one layer of icing.



Each layer of cake has radius 10 cm and height 3 cm.
The icing, between the two layers of cake, has radius 10 cm and height 12 mm.

(a) Calculate the volume of **icing** in the birthday cake.
Give your answer in cm^3.

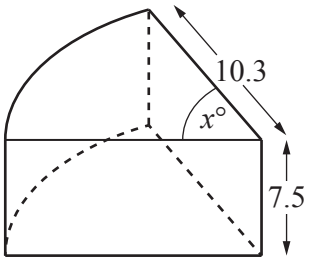
..... cm^3 [2]

(b) The top and curved surface of the birthday cake are now covered with chocolate.

Calculate the area of the birthday cake that is covered with chocolate.

..... cm^2 [3]

(c) Anil has a slice of this chocolate-covered birthday cake.



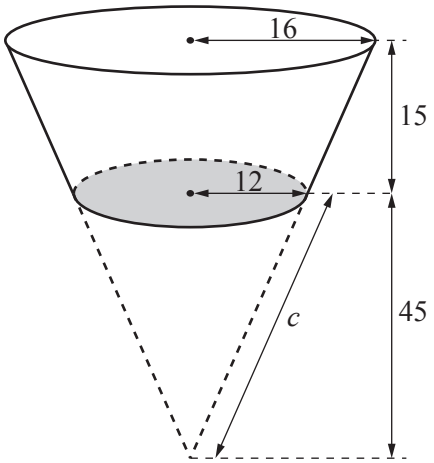
His slice is a prism of height 7.5 cm.
The top of the cake is a sector, radius 10.3 cm and angle $x°$.
The volume of his slice is 200 cm^3.

Calculate the value of x.

$x =$ ..... [3]

11 **[Volume of cone = $\frac{1}{3}\pi r^2 h$]**

[Curved surface area of a cone = $\pi r l$]



The diagram shows a bowl with a circular base.
The curved surface of the bowl is formed by removing a cone with radius 12 cm and height 45 cm from a larger cone as shown in the diagram.
The radius of the top of the bowl is 16 cm and its height is 15 cm.

(a) Calculate the volume of the bowl.

..... cm^3 [3]

(b) The slant height of the cone that has been removed is c cm.

Show that $c = 46.6$, correct to 3 significant figures.

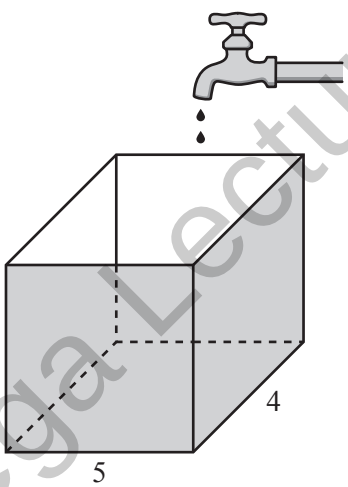
[2]

(c) The bowl is completely filled with water.

Calculate the total surface area of the bowl that is in contact with the water.

..... cm^2 [4]

12



Water drips from a tap into a container which stands on a horizontal surface.
The container is a cuboid with base 5 cm by 4 cm.

The volume of **each** drop of water is 0.08 cm^3.

Calculate the change in water level caused by 400 drops.

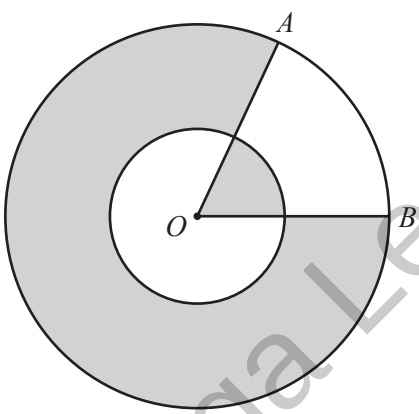
..... cm [3]

13 A cuboid has a square base.
The length of the base of the cuboid is y cm.
The height of the cuboid is twice the length of its base.
The total surface area of the cuboid is $360\,\text{cm}^2$.

Find the height of the cuboid.

..... cm [3]

14



NOT TO SCALE

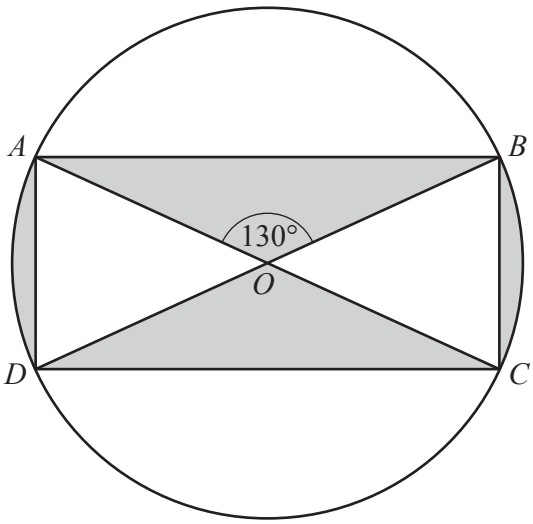
The diagram shows two circles, both with centre O.
The radius of the small circle is 3 cm and the radius of the large circle is 6 cm.
The minor sector AOB has an angle of 60°.

The total area of the shaded regions is $k\pi\,\text{cm}^2$.

Find the value of k.

$k =$ ..... [4]

15



AC and BD are diameters of the circle, centre O.
$AC = 12$ cm and $A\hat{O}B = 130°$.

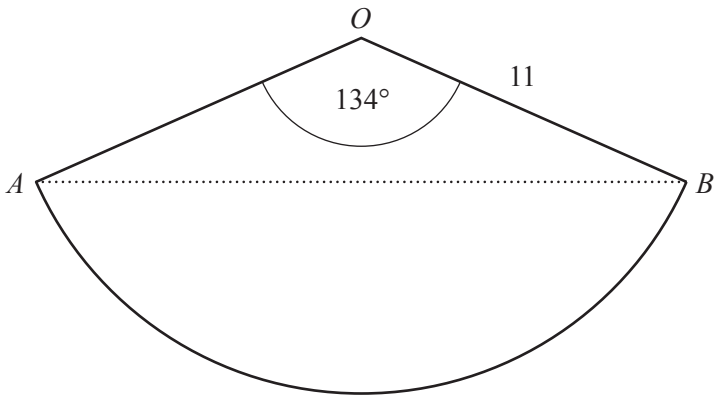
(a) Calculate the area of triangle AOB.

Answer ..... cm^2 [2]

(b) Calculate the area of the sector AOD.

Answer ..... cm^2 [2]

16 (a)



OAB is a sector of a circle, centre O, radius 11 cm.
$A\hat{O}B = 134°$.

(i) Calculate the length of the arc AB.

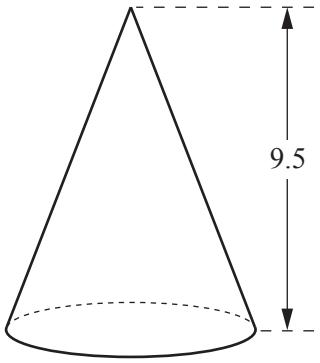
Answer ..... cm [2]

(ii) Calculate the shortest distance from O to the line AB.

Answer ..... cm [2]

(b) **[Volume of a cone = $\frac{1}{3}\pi r^2 h$]**

[Curved surface area of a cone = πrl]



A cone has height 9.5 cm and volume 115 cm^3.

(i) Show that the radius of the base of the cone is 3.4 cm, correct to 1 decimal place.

[2]

(ii) Calculate the curved surface area of the cone.

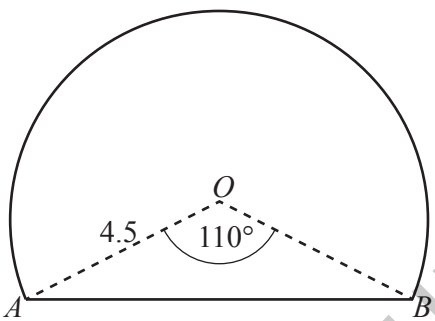
Answer ..... cm^2 [3]

17 **(a)** The ventilation shaft for a tunnel is in the shape of a cylinder.
The cylinder has radius 0.4 m and length 15 m.

Calculate the volume of the cylinder.

Answer ..... m^3 [2]

(b) The diagram shows the cross-section of the tunnel.

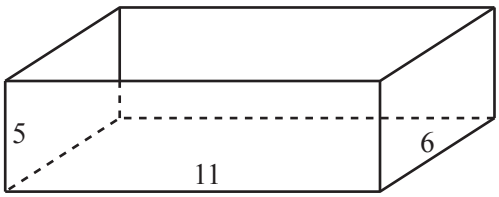


The cross-section of the tunnel is a major segment of a circle, centre O.
The radius of the circle is 4.5 m and $A\hat{O}B = 110°$.

Calculate the area of the cross-section of the tunnel.

Answer ..... m^2 [4]

18



An **open** rectangular tray has inside measurements

length 11 cm width 6 cm height 5 cm.

(a) Calculate the total surface area of the four sides and base of the inside of the tray.

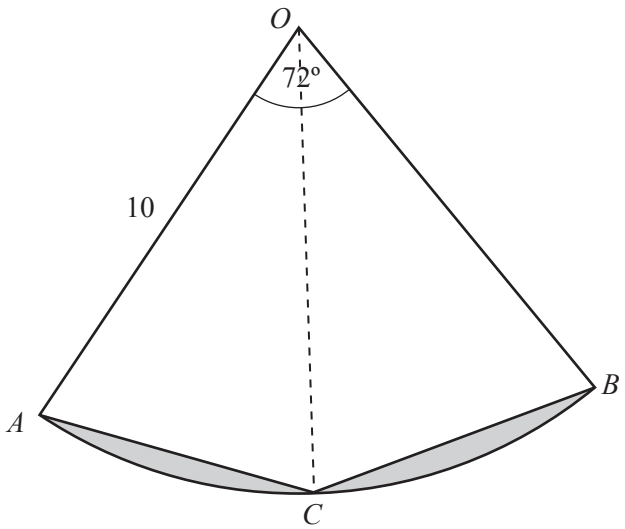
Answer ..... cm^2 [2]

(b) Cubes are placed in the tray and a lid is placed on top.
Each cube has an edge of 2 cm.

Find the maximum number of cubes that can be placed in the tray.

Answer ..... [1]

19



OAB is a sector of a circle, centre *O*, and radius 10 cm.
$A\hat{O}B = 72°$ and *C* is the point on the arc *AB* such that *OC* bisects $A\hat{O}B$.

(a) Calculate the perimeter of sector *OAB*.

..... cm [3]

(b) (i) Calculate the area of sector *OAB*.

..... cm^2 [2]

(ii) Calculate the total shaded area.

..... cm^2 [3]

20 **[The volume of a sphere is $\frac{4}{3}\pi r^3$]**

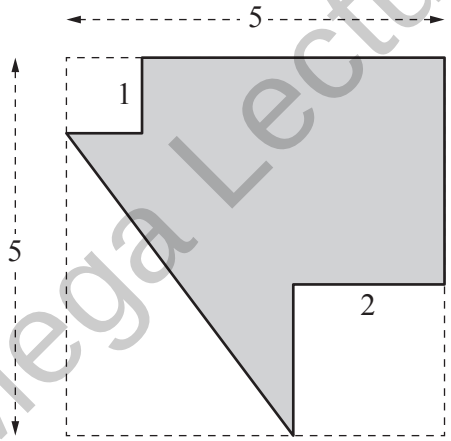
During a storm, raindrops fall into a cylinder which stands on horizontal ground.
The cylinder was empty before the storm started.

The cylinder has radius 20 mm.
Each raindrop is a sphere of radius 2 mm.
After the storm, the depth of water in the cylinder is 16 mm.

Calculate the number of raindrops that fell into the cylinder.

Answer ..... [3]

21



The diagram shows a square piece of card, from which a triangle and two small squares are removed.
All lengths on the diagram are in centimetres.

(a) Calculate the area of the shaded card.

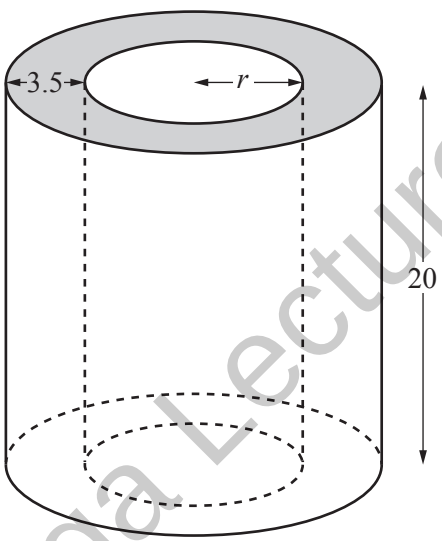
Answer ..... cm^2 [2]

(b) Calculate the perimeter of the shaded card.

Answer ..... cm [2]

22 **[Volume of a cone = $\frac{1}{3}\pi r^2 h$]**

(a)



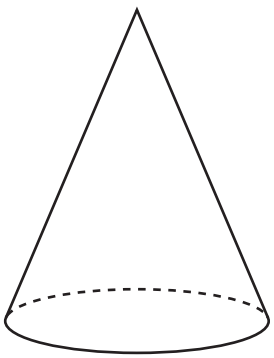
Solid I

Solid I is a cylinder with a small cylinder removed from its centre, as shown in the diagram.
The height of each cylinder is 20 cm and the radius of the small cylinder is r cm.
The radius of the large cylinder is 3.5 cm greater than the radius of the small cylinder.
The volume of Solid I is 3000 cm^3.

(i) Calculate r.

Answer $r =$ ..... [4]

(ii) Solid II is a cone with volume of 3000 cm^3.
The perpendicular height of the cone is twice its radius.

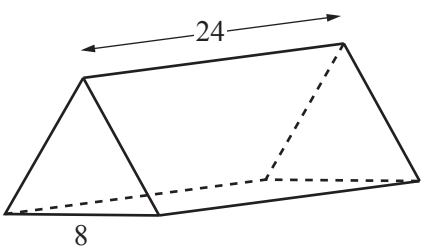


Solid II

Which solid is the taller and by how much?

Answer Solid ..... is the taller by ..... cm [4]

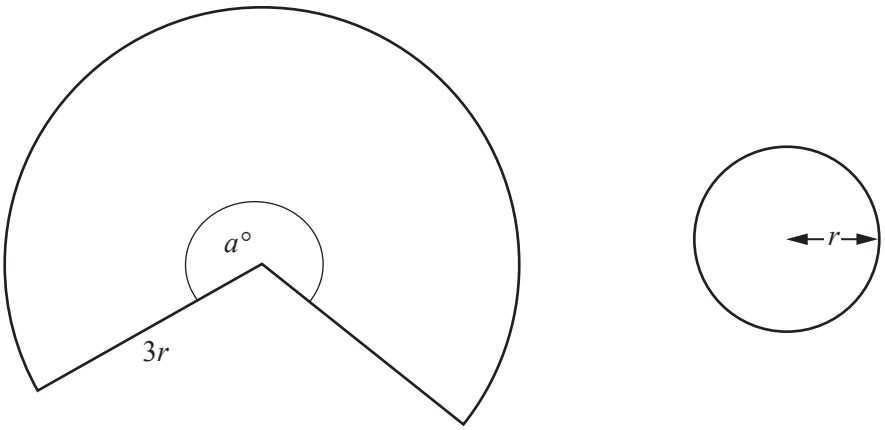
(b) The diagram shows a triangular prism of length 24 cm.
Its cross-section is an equilateral triangle with sides 8 cm.



Calculate the **total** surface area of the prism.

Answer ..... cm^2 [4]

23



The diagram shows a sector of a circle with radius $3r$ cm and angle $a°$ and a circle with radius r cm.
The ratio of the area of the sector to the area of the circle with radius r cm is 8 : 1.

(a) Find the value of a.

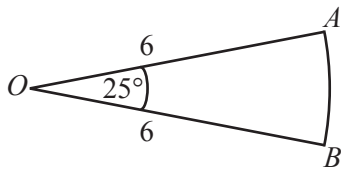
Answer $a =$ ..... [3]

(b) Find an expression, in terms of π and r, for the perimeter of the sector.

Answer ..... cm [2]

24 **(a)** OAB is a sector of a circle, centre O, radius 6 cm.

$A\hat{O}B = 25°$.



(i) Calculate the length of the arc AB.

Answer ..... cm [2]

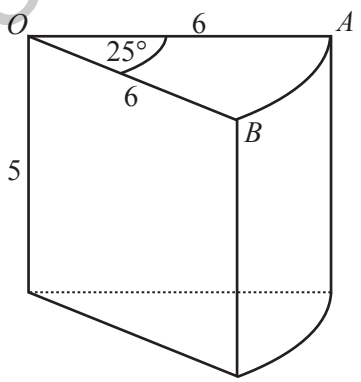
(ii) Calculate the area of the sector OAB.

Answer ..... cm^2 [2]

(b) The sector OAB from part **(a)** is the cross-section of a slice of cheese.

The slice has a height of 5 cm.

(i) Calculate the volume of this slice of cheese.



Answer ..... cm^3 [1]

(ii) Calculate the total surface area of this slice of cheese.

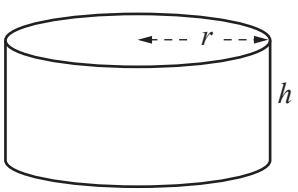
Answer ..... cm^2 [3]

(iii) Another 25° slice of cheese has 3 times the height and twice the radius.

Calculate its volume.

Answer ..... cm^3 [2]

(c) A dairy produces cylindrical cheeses, each with a volume of 800 cm^3.



The height h cm and the radius r cm can vary.

(i) Express h in terms of r.

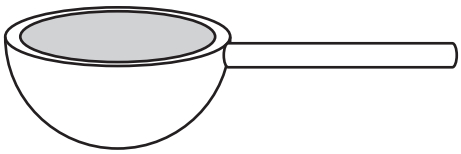
Answer ..... [1]

(ii) What happens to the height if the radius is doubled?

Answer ..... [1]

25 **[The volume of a sphere is $\frac{4}{3}\pi r^3$]**

(a)



A spoon used for measuring in cookery consists of a hemispherical bowl and a handle.
The internal volume of the hemispherical bowl is 20 cm^3.
The handle is of length 5 cm.

(i) Find the internal radius of the hemispherical bowl.

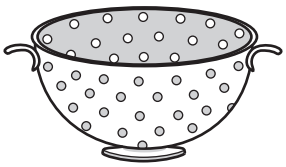
Answer ..... cm [2]

(ii) The hemispherical bowl of a geometrically similar spoon has an internal volume of 50 cm^3.

Find the length of its handle.

Answer ..... cm [2]

(b) [The surface area of a sphere is $4\pi r^2$]

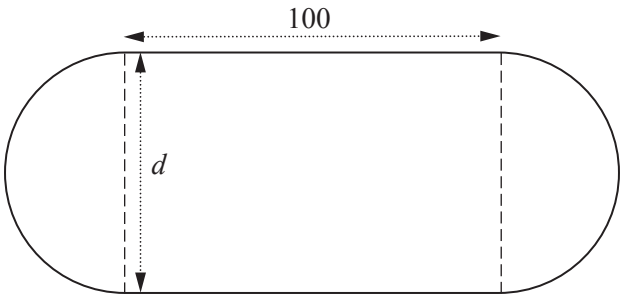


An open hemisphere of radius 5.5 cm is used to make a metal kitchen strainer.
50 holes are cut out of the curved surface.
Assume that the piece of metal removed to make each hole is a circle of radius 1.5 mm.

Calculate the external surface area that remains.

Answer ..... cm^2 [3]

26



The diagram shows the perimeter of a 400 m running track.
It consists of a rectangle measuring 100 m by d metres and two semicircles of diameter d metres.
The length of each semicircular arc is 100 m.

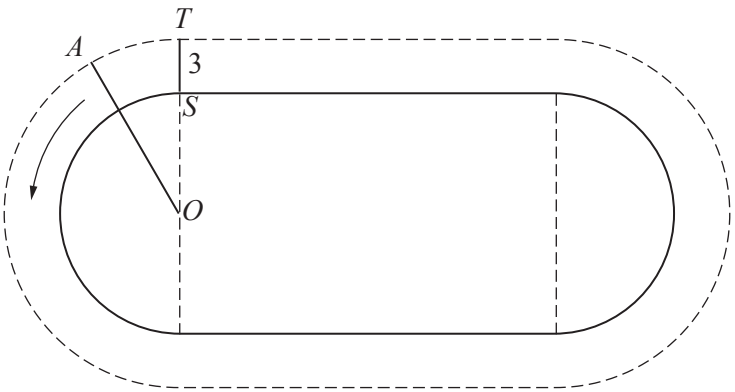
(a) Calculate d.

Answer $d =$ ..... [2]

(b) Calculate the total area of the region inside the running track.

Answer ..... m^2 [3]

(c)



S is the starting point and finishing point for the 400 m race for a runner in the inside lane.
A runner in an outer lane is always 3 m from the inner perimeter.
The runner in the outer lane starts at *A*, runs 400 m and finishes at *T*.
TS = 3 m.

(i) Calculate the length of the arc *TA*.

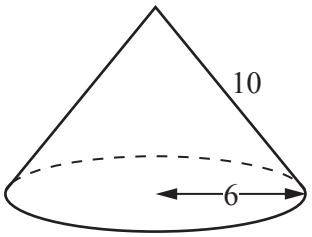
Answer ..... m [3]

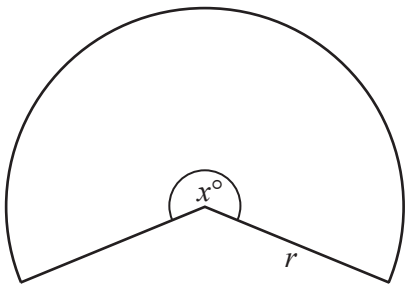
(ii) *O* is the centre of a semi-circular part of the track.

Calculate $A\hat{O}T$.

Answer $A\hat{O}T$ = ... [2]

27





A hollow cone has a base radius 6 cm and slant height 10 cm.
The curved surface of the cone is cut, and opened out into the shape of a sector of a circle, with angle $x°$ and radius r cm.

(a) Write down the value of r.

Answer $r =$ ..... [1]

(b) Calculate x.

Answer $x =$ ..... [2]

28 [The volume of a sphere is $\frac{4}{3}\pi r^3$]

20 spheres, each of radius 3 cm, have a total volume of $k\pi$ cm^3.

(a) Find the value of k.

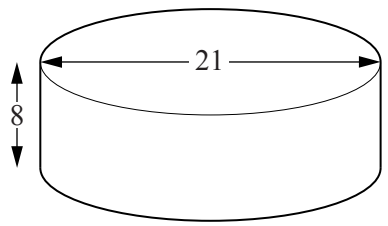
Answer $k =$ ..... [1]

(b) The spheres are inside an open cylinder, with radius 6 cm.
The cylinder stands on a horizontal surface and contains enough water to cover the spheres.

Calculate the change in depth of the water when the spheres are taken out of the cylinder.

Answer ..... cm [2]

29 A cylindrical, **open** container has a diameter of 21 cm and height of 8 cm.



(a) (i) Calculate the **total** external surface area of this container.

Answer ..... cm^2 [3]

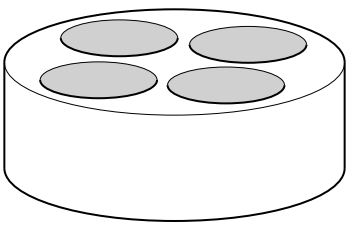
(ii) A manufacturer receives an order for 30 000 containers.
He needs an extra 150 cm^2 of material for each container to cover wastage.

Calculate the area of material needed to make these containers.
Give your answer in square metres.

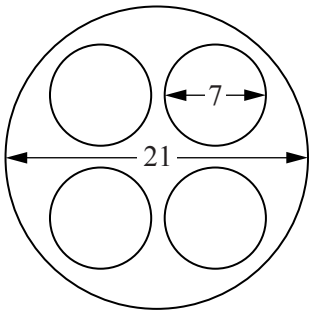
..... m^2 [2]

[The Surface area of a sphere is $4\pi r^2$] [The Volume of a sphere is $\frac{4}{3}\pi r^3$]

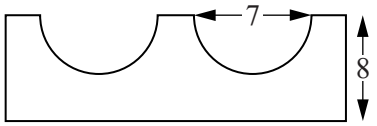
(b) A circular top that can hold 4 hemispherical bowls can be placed on the container.



Container and Top



Top



Cross-section

The top is a circle of diameter 21 cm with four circular holes of diameter 7 cm.
A hemispherical bowl of diameter 7 cm fits into each hole.
The cross-section shows two of these bowls.

Calculate the inside curved surface area of one of these hemispherical bowls.

..... cm^2 [1]

Calculate the total surface area of the top of the container, including the inside curved surface area of each bowl.

..... cm^2 [3]

With the top and the 4 bowls in place, calculate the volume of water required to fill the container.

.....cm^3 [3]