Cambridge Assessment International Education

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4908419373

ADDITIONAL MATHEMATICS **0606/23**

Paper 2 **October/November 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Solve $|3x-2| = 4 + x$. [3]

$3x - 2 = 4 + x$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

$3x - 2 = -(4 + x)$

$3x - 2 = -4 - x$

$\frac{4x}{4} = \frac{-2}{4}$

$x = -\frac{1}{2}$ or -0.5

2 Solve the simultaneous equations.

$$x^2 + 3xy = 4$$
$$2x + 5y = 4$$

[5]

$2x + 5y = 4$

$\frac{5y}{5} = \frac{4-2x}{5}$

$y = \frac{4-2x}{5}$

$x^2 + 3x\left(\frac{4-2x}{5}\right) = 4$

$5x^2 + 12x - 6x^2 = 20$

$5x^2 + 12x - 6x^2 = 20$

$-x^2 + 12x - 20 = 0$

$x^2 - 12x + 20 = 0$

$(x-2)(x-10) = 0$

$x - 2 = 0$ $\quad x - 10 = 0$

$x = 2$ $\quad x = 10$

$y = \frac{4-2x}{5}$

$y = \frac{4-2(2)}{5}$

$y = \frac{0}{5}$ $\quad y = 0$

$y = \frac{4-2x}{5}$

$y = \frac{4-2(10)}{5}$

$y = \frac{4-20}{5}$

$y = -\frac{16}{5}$

$y = -3.2$

$x = 10, \quad y = -3.2$

$x = 2, \quad y = 0$

3 Find the values of k for which the equation $x^2+(k+9)x+9=0$ has two distinct real roots. [4]

$(k+9)^2 - 4(1)(9) > 0$

$(k+9)^2 - 36 > 0$

$(k+9)^2 - 36 = 0$

$(k+9)^2 = 36$

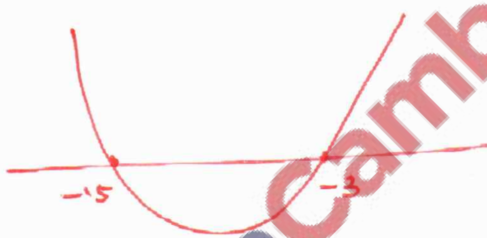
$(k+9)^2 = 36$

$k+9 = \sqrt{36}$

$k+9 = \pm 6$

$k = 6-9$ | $k = -6-9$

$k = -3$ | $k = -15$



$k < -15$

$k > -3$

4 It is given that $y=\ln(1+\sin x)$ for $0<x<\pi$.

(a) Find $\frac{dy}{dx}$. [2]

$\frac{dy}{dx} = \frac{1}{1+\sin x} \cdot \cos x$

$= \frac{\cos x}{1+\sin x}$

(b) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$, giving your answer in the form $\frac{1}{\sqrt{a}}$, where a is an integer. [2]

$\frac{dy}{dx} = \frac{\cos x}{1+\sin x}$

$x = \frac{180^\circ}{6} = 30^\circ, 45^\circ, 60^\circ$

$\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\frac{\cos 30^\circ}{1+\sin 30^\circ} = \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}$

$= \frac{\sqrt{3}}{2} \times \frac{2}{3} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$= \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}} \times \sqrt{3}} = \frac{1}{\sqrt{3}}$

(c) Find the values of x for which $\frac{dy}{dx} = \tan x$. [5]

$\tan x = \frac{\sin x}{\cos x}$

$\frac{\cos x}{1+\sin x} = \tan x$

$\frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x}$

$\cos^2 x = \sin x + \sin^2 x$

$1 - \sin^2 x = \sin x + \sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ $\quad$ $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -1$

$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ $\quad$ $x =$ Invalid

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

5 Solve the following simultaneous equations.

$$3^x \times 9^{y-1} = 243$$

$$8 \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{4\sqrt{2}}$$ [5]

$3^x + 3^{2(y-1)} = 3^5$

$3^x + 3^{2y-2} = 3^5$

$x + 2y - 2 = 5$

$x + 2y = 7$ - (i)

$2^3 \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{2^x \times 2^{\frac{1}{2}}}$

$\cancel{2}^{3+y-\frac{1}{2}} = \cancel{2}^{2x+1-(2.5)}$

$2.5 + y = 2x - 1.5$

$y = 2x - 1.5 - 2.5$

$y = 2x - 4$ - (ii)

$x + 2y = 7$

$y = 2x - 4$

$x + 2(2x-4) = 7$

$x + 4x - 8 = 7$

$\frac{5x}{5} = \frac{15}{5}$

$x = \underline{3}$

$y = 2x - 4$

$y = 2(3) - 4$

$y = 6 - 4$

$y = \underline{2}$

6 A 4-digit code is to be formed using 4 different numbers selected from 1, 2, 3, 4, 5, 6, 7, 8 and 9. Find how many different codes can be formed if

(a) there are no restrictions, [1]

$^9P_4 = \underline{\underline{3024}}$

$9 \times 8 \times 7 \times 6 = \underline{\underline{3024}}$

(b) only prime numbers are used, [1]

1 2 3 4 5 6 7 8 9

$4 \times 3 \times 2 \times 1 = \underline{\underline{24}}$

(c) two even numbers are followed by two odd numbers, [2]

1 2 3 $\cancel{4}$ 5 6 7 8 9

$4 \times 3 \times 5 \times 4$

$= \underline{\underline{240}}$

(d) the code forms an even number. [2]

1 $\cancel{2}$ 3 $\cancel{4}$ 5 $\cancel{6}$ 7 $\cancel{8}$ 9

$= 8 \times 7 \times 6 \times 4$

$= \underline{\underline{1344}}$

7 A curve has equation $y = x\cos x$.

(a) Find $\dfrac{dy}{dx}$. [2]

$\dfrac{dy}{dx} = x \cdot -\sin x + \cos x \cdot 1$

$= \cos x - x\sin x$

$= \cos x - x\sin x$

(b) Find the equation of the normal to the curve at the point where $x = \pi$, giving your answer in the form $y = mx + c$. [4]

Gradient = Normal × tangent = −1

$M_N = \dfrac{-1}{M_t}$

$Y = x\cos x$

$Y = \pi\cos\pi$

$= -\pi$

$\begin{pmatrix} \pi, & -\pi \\ x, & y \end{pmatrix}$

$\dfrac{y+\pi}{x-\pi} = \dfrac{-1}{1}$

$y + \pi = -x + x$

$y = -x + 0$

$y = mx + c$

$y = \underline{-x + 0}$

(c) Using your answer to **part (a)**, find the exact value of $\int_0^{\frac{\pi}{6}} x \sin x \, dx$. [5]

$\frac{dy}{dx} = \cos x - x \sin x$

$\int \cos x - x \sin x \, dx = y = x \cos x$

$\int \cos x \, dx - \int x \sin x \, dx = x \cos x$

$\int x \sin x \, dx = \int \cos x \, dx - x \cos x$

$\int_0^{\pi/6} x \sin x \, dx = \left[\int_0^{\pi/6} \cos x \, dx\right] - \left[x \cos x\right]_0^{\pi/6}$

$= \left[\sin x\right]_0^{\pi/6} - \left[x \cos x\right]_0^{\pi/6}$

$= \left[\sin x - x \cos x\right]_0^{\pi/6}$

$= \left[\sin \frac{\pi}{6} - \frac{\pi}{6} \cos \frac{\pi}{6}\right] - \left[\sin 0 - 0\right]$
$= 0$

$= \frac{1}{2} - \frac{\pi}{6} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{2}\left(1 - \frac{\sqrt{3}\pi}{6}\right)$

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

$$\log_2(y+1) = 3 - 2\log_2 x$$
$$\log_2(x+2) = 2 + \log_2 y$$

(a) Show that $x^3 + 6x^2 - 32 = 0$. [4]

$\log_2(y+1) + 2\log_2 x = 3$

$\log_2(y+1) + \log_2 x^2 = 3$

$\log_2 (y+1)\cdot x^2 = 3$

$(y+1)x^2 = 2^3$

$(y+1)x^2 = 8$ ----(i)

$\log_2(x+2) - \log_2 y = 2$

$\log_2 \frac{x+2}{y} = 2$

$\frac{x+2}{y} = 2^2$

$x+2 = 4y$ ---(ii)

$\frac{4y}{4} = \frac{x+2}{4}$

$\left(\frac{x+2}{4} + 1\right)x^2 = 8$

$\left(\frac{x+2+4}{\cancel{4}}\right)x^2 = 8 \times 4$

$(x+6)x^2 = 32$

$\underline{x^3 + 6x^2 - 32 = 0}$

(b) Find the roots of $x^3 + 6x^2 - 32 = 0$. [4]

Let $f(x) = x^3 + 6x^2 - 32$

$f(2) = 8 + 6(4) - 32 = 0$

$x=2$, $f(x) = 0$ $\underline{\underline{= 0}}$

$(x-2)$

$$\begin{array}{r|l} & x^2 + 8x + 16 \\ \hline x-2 & x^3 + 6x^2 + 0x - 32 \\ & x^3 - 2x^2 \\ \hline & 8x^2 + 0x - 32 \\ & 8x^2 - 16x \\ \hline & 16x - 32 \\ & 16x - 32 \\ \hline & 0 \end{array}$$

$f(x) = (x-2)(x^2+8x+16)$

$f(x) = (x-2)(x+4)(x+4)$

$(x-2)(x+4)(x+4) = 0$

$x = \underline{\underline{2}}$, $x = -\underline{\underline{4}}$ $x = -\underline{\underline{4}}$

(c) Give a reason why only one root is a valid solution of the logarithmic equations. Find the value of y corresponding to this root. [2]

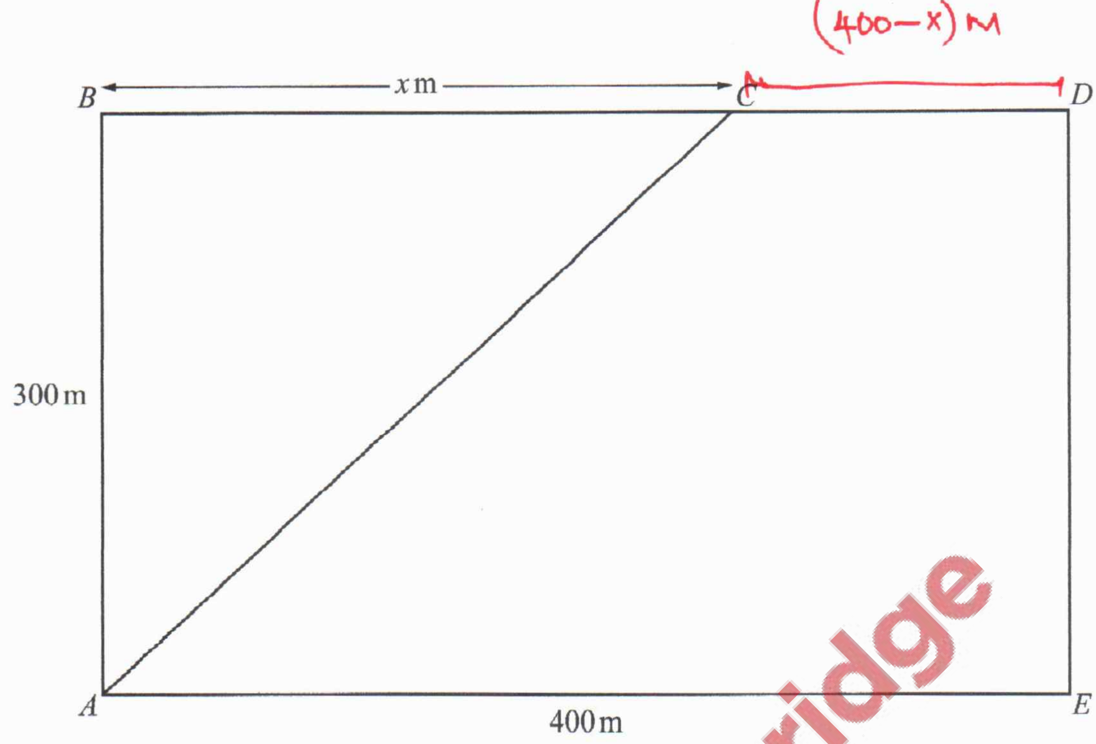
$x = 2$, $x = -4$

$x = 2$, log cannot take negative values.

equation (2) is not valid.

$y = 1$

9



The rectangle $ABCDE$ represents a ploughed field where $AB = 300\,\text{m}$ and $AE = 400\,\text{m}$. Joseph needs to walk from A to D in the least possible time. He can walk at $0.9\,\text{ms}^{-1}$ on the ploughed field and at $1.5\,\text{ms}^{-1}$ on any part of the path BCD along the edge of the field. He walks from A to C and then from C to D. The distance $BC = x\,\text{m}$.

(a) Find, in terms of x, the total time, T s, Joseph takes for the journey. [3]

$AC^2 = x^2 + 300^2$

$AC^2 = x^2 + 90{,}000$

$AC = \sqrt{90{,}000 + x^2}$

speed $= D/t$

$T = \dfrac{d}{s}$

$T = \dfrac{\sqrt{90{,}000 + x^2}}{0.9} + \dfrac{400 - x}{1.5} =$

$= \dfrac{1}{0.9}\left(90{,}000 + x^2\right)^{\frac{1}{2}} + \dfrac{1}{1.5}(400 - x)$

$\dfrac{dT}{dx} = \dfrac{1}{0.9} \cdot \dfrac{1}{2}\left(90{,}000 + x^2\right)^{-\frac{1}{2}} \cdot 2x + \dfrac{1}{1.5}(0 - 1)$ $T =$

$= \dfrac{x}{0.9\left(90{,}000 + x^2\right)^{\frac{1}{2}}} - \dfrac{1}{1.5} = 0$

$\dfrac{x}{0.9\left(90{,}000 + x^2\right)^{\frac{1}{2}}} = \dfrac{1}{1.5}$

$1.5x = 0.9\left(90{,}000 + x^2\right)^{\frac{1}{2}}$

$= 2.25x^2 = 0.81\left(90{,}000 + x^2\right)$

$2.25x^2 = 72900 + 0.81x^2$

$\dfrac{\cancel{1.44}x^2}{\cancel{1.44}} = \dfrac{72900}{1.44}$

$x^2 = \sqrt{50625}$

$x = \underline{\underline{225}}$

Total time $=$

$T = \dfrac{90{,}000 + x^2}{0.9} + \dfrac{400 - x}{1.5}$

(b) Given that x can vary, find the value of x for which T is a minimum and hence find the minimum value of T. [6]

$$T = \left(\frac{\sqrt{90{,}000 + x^2}}{0.9} + \frac{400 - x}{1.5}\right) s$$

$$= \frac{1}{0.9}\left(90{,}000 + x^2\right)^{\frac{1}{2}} + \frac{1}{1.5}\left(400 - x\right)$$

$$\frac{dT}{dx} = \frac{1}{0.9} \cdot \frac{1}{2}\left(90{,}000 + x^2\right)^{-\frac{1}{2}} \cdot 2x + \frac{1}{1.5}(0-1)$$

$$= \frac{x}{0.9\left(90{,}000 + x^2\right)^{\frac{1}{2}}} - \frac{1}{1.5}$$

$$\frac{x}{0.9\left(90{,}000 + x^2\right)^{\frac{1}{2}}} = \frac{1}{1.5}$$

$$1.5x = 0.9\left(90{,}000 + x^2\right)^{\frac{1}{2}}$$

$$2.25x^2 = 0.81\left(90{,}000 + x^2\right)$$

$$2.25x^2 = 72900 + 0.81x^2$$

$$\frac{\cancel{1.44}x^2}{\cancel{1.44}} = \frac{72900}{x^2}$$

$$x^2 = \sqrt{50625}$$

$$x = \underline{\underline{225}}$$

$$T = \frac{\sqrt{90{,}000 + 50625}}{0.9} + \frac{400 - 225}{1.5}$$

$$T = 416\tfrac{2}{3} + 116\tfrac{2}{3}$$

$$T = 533.333$$

$$T = \underline{\underline{533\tfrac{1}{3} \text{ seconds}}}$$

10 (a) The sum of the first 4 terms of an arithmetic progression is 38 and the sum of the next 4 terms is 86. Find the first term and the common difference. [5]

$U_n = a+(n-1)d$

$S_4 = 38$

$n=4$

$2(2a+3d)=38$

$2a+3d=19$ -(i)

$S_4 = 38$

$S_8 - S_4 = S_{next\ 4\ term}$

$= 86$

$S_8 = 86 + S_4 = 86+38$

$= \underline{\underline{124}}$

$n=8$

$\frac{4}{4}(2a+7d) = \frac{124}{4}$

$2a+7d = 31$

$2a = 31-7d$ --- (2)

$31-7d+3d=19$

$-4d = -12$

$d = \underline{\underline{3}}$

$2a = 31-7(\;)$

$2a = 31-21$

$\frac{2a}{2} = \frac{10}{2}$

$a = \underline{\underline{5}}$

First term (a) = 5

Common difference (d) = 3

(b) The third term of a geometric progression is 12 and the sixth term is -96. Find the sum of the first 10 terms of this progression. [6]

$U^n = ar^{n-1}$

$S_\infty = \frac{a}{1-r} = |r| < 1$

$U_3 = 12$

$n=3$

$ar^2 = 12 \quad \text{--}(i)$

$S_n = \frac{a(1-r^n}{1-r}$

$U_6 = -96$

$n=6 \quad ar^5 = -96 \; \text{-}(ii)$

$\frac{\cancel{a}r^5}{\cancel{a}r^2} = \frac{-96}{12}$

$r^3 = -8$

$r = \sqrt[3]{-8}$

$r = \underline{\underline{-2}}$

$a = \frac{12}{4}$

$a = \underline{\underline{3}}$

$a = \underline{\underline{3}}, \; r = \underline{\underline{-2}}$

$S_{10} = \frac{3(1-(-2)^{10})}{1 - -(2)}$

$= \frac{\cancel{3}(1-1024)}{\cancel{3}}$

$= \underline{\underline{-1023}}$

Question 11 is printed on the next page.

11 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the quadratic equation $(\sqrt{7}-2)x^2 - 4x + (\sqrt{7}+2) = 0$, giving each of your answers in the form $a + b\sqrt{7}$, where a and b are constants. [7]

$$x = \frac{4 \pm \sqrt{16 - 4(\sqrt{7}-2)(\sqrt{7}+2)}}{2(\sqrt{7}-2)}$$

$$x = \frac{4 \pm \sqrt{16 - 4(3)}}{2(\sqrt{7}-2)}$$

$$x = \frac{4 \pm \sqrt{4}}{2(\sqrt{7}-2)}$$

$$x = \frac{4 \pm \sqrt{4}}{2(\sqrt{7}-2)}$$

$$x = \frac{2+1}{\sqrt{7}-2} \quad \text{or} \quad \frac{2-1}{\sqrt{7}-2}$$

$$x = \frac{3}{\sqrt{7}-2} \times \frac{(\sqrt{7}+2)}{(\sqrt{7}+2)}$$

$$= \frac{3\sqrt{7}+6}{3}$$

$$x = \underline{\underline{2+\sqrt{7}}}$$

$$\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$$

$$\frac{\sqrt{7}+2}{3}$$

$$\frac{2+\sqrt{7}}{3}$$

$$x = \underline{\underline{\frac{2}{3} + \frac{1}{3}\sqrt{7}}}$$