Series Past Papers 2002-2019

Question 1:

A progression has a first term of 12 and a fifth term of 18.

(i) Find the sum of the first 25 terms if the progression is arithmetic. [3]

(ii) Find the 13th term if the progression is geometric. [4]

Answer:

4 (i) $a=12$ $a+4d=18$ $\therefore d=1.5$	B1	Correct only
$S_{25}=25/2(24+24\times1.5)$ $=750$	M1 A1 3	Use of S_n formula. Correct only.
(ii) $a=12$ $ar^4=18$ $r^4=1.5$ 13 th term $=ar^{12}$	M1 A1	Correct method for r or r^4 (needs ar^4)
$=12\times(1.5)^3$	M1	Needs ar^{12} and method for subbing r (or r^4)
$=40.5$ or 40.6	A1 4	Correct only.

9709/1/M/J/02

Question 2:

A geometric progression, for which the common ratio is positive, has a second term of 18 and a fourth term of 8. Find

(i) the first term and the common ratio of the progression, [3]

(ii) the sum to infinity of the progression. [2]

Answer

2. (i) $ar=18$ and $ar^3=8$	M1	Any 2 equations of type ar^n
Solution to give $r=2/3$	DM1	Correct method on correct 2 equations.
$a=18\div r=27.0$	A1 3	For his $18\div r$
(ii) Sum to infinity $=a\div(1-r)$	M1	Correct formula applied – even if $r>1$.
Answer $=81.0$	A1√ 2	Follow through provided $r<1$.
		(ignore $r=\pm2/3$)

9709/1/O/N/02

Question 3

(a) A debt of \$3726 is repaid by weekly payments which are in arithmetic progression. The first payment is \$60 and the debt is fully repaid after 48 weeks. Find the third payment. [3]

(b) Find the sum to infinity of the geometric progression whose first term is 6 and whose second term is 4. [3]

Answer

3 (a) $a=60$, $n=48$, $S_n=3726$		
S_n formula used	M1	Correct formula (M0 if nth term used)
$\rightarrow d=\$0.75$	A1	Co
3^{rd} term $=a+2d=\$61.50$	A1√ [3]	Use of a+2d with his d. 61.5 ok.
(b) $a=6$ $ar=4$ $\therefore r=⅔$	M1	a, ar correct, and r evaluated
$S_\infty=a/(1-r)=18$	M1A1 [3]	Correct formula used, but needs $\lvert r\rvert<1$ for M mark

9709/01/O/N/03

Question 4:

Find

(i) the sum of the first ten terms of the geometric progression 81, 54, 36, … , [3]

(ii) the sum of all the terms in the arithmetic progression 180, 175, 170, … , 25. [3]

Answer

(i) 81,54,36 r = 54/81 or 36/54	B1	Value of r – unsimplified – allow 0.66
$S_{10} = 81\,(1-⅔^{10}) \div (1-⅔)$	M1	Correct formula – power 10 and used
$\rightarrow 239$	A1	Co. More than 3 s.f. ok, but needs 238.8
	[3]	
(ii) n = (180 – 25) ÷ 5 + 1 = 32	B1	31 gets M0
Use of any S_n formula	M1	Correct formula – not for n = 25,5,180
$\rightarrow 3280$	A1	Co
	[3]	

9709/01/O/N/04

Question 5:

A geometric progression has first term 64 and sum to infinity 256. Find

(i) the common ratio, [2]

(ii) the sum of the first ten terms. [2]

Answer

(i) a/(1–r) = 256 and a = 64	M1	Use of correct formula
$\rightarrow$ r = ¾	A1	Correct only
	[2]	
(ii) $S_{10} = 64(1-0.75^{10})\ \ (1-0.75)$	M1	Use of correct formula – 0.75^{10} not 0.75^9
$\rightarrow S_{10} = 242$	A1	Correct only
	[2]	

9709/01/M/J/04

Question 6:

A geometric progression has 6 terms. The first term is 192 and the common ratio is 1.5. An arithmetic progression has 21 terms and common difference 1.5. Given that the sum of all the terms in the geometric progression is equal to the sum of all the terms in the arithmetic progression, find the first term and the last term of the arithmetic progression. [6]

Answer

GP $a = 192,\ r = 1.5,\ n = 6$		
AP $a = a,\ d = 1.5,\ n = 21$		
S_6 for GP $= 192(1.5^6 - 1) \div 0.5$	**M1**	Correct sum formula used.
$= 3990$		
S_{21} for AP $= \dfrac{21}{2}(2a + 20 \times 1.5)$	**M1**	Correct sum formula used.
	DM1	Needs both M's - soln of sim eqns.
	A1	CAO
Equate and solve $\rightarrow a = 175$		
21^{st} term in AP $= a + 20d = 205$	**M1 A1**	Correct formula used.
(or from $3990 = 21(a + l)/2$	**[6]**	

9709/01/M/J/05

Question 7:

A small trading company made a profit of \$250 000 in the year 2000. The company considered two different plans, plan A and plan B, for increasing its profits.

Under plan A, the annual profit would increase each year by 5% of its value in the preceding year. Find, for plan A,

(i) the profit for the year 2008, [3]

(ii) the total profit for the 10 years 2000 to 2009 inclusive. [2]

Under plan B, the annual profit would increase each year by a constant amount \$$D$.

(iii) Find the value of D for which the total profit for the 10 years 2000 to 2009 inclusive would be the same for both plans. [3]

Answer

(i) GP with $a = 250000$ $r = 1.05$ Year 2008 is the 9th term $ar^8 = 250000 \times 1.05^8 = 369\ 000$	B1 M1 A1√ [3]	For any use of r=1.05 (25000 + 0.05 x 25000 gets B1) Use of ar^{n-1} with r=8 or 9 Answer rounding to 369 000. ft on *r*.
(ii) $S_{10} = 250000(1.05^{10} - 1) \div 0.05$ $= 3\ 140\ 000$ (iii) AP $S_{10} = 5(500\ 000 + 9D)$ = answer to (ii) $\rightarrow D = 14\ 300$	M1 A1 [2] M1 DM1 A1 [3]	Use of correct S_n formula – for 10 only Co – must round to the correct answer (adds 10 numbers correctly M1 A1) Correct S_n formula. Forming + soln Co.

9709/01/O/N/05

Question 8:

(a) Find the sum of all the integers between 100 and 400 that are divisible by 7. [4]

(b) The first three terms in a geometric progression are 144, x and 64 respectively, where x is positive. Find

(i) the value of x,

(ii) the sum to infinity of the progression.

[5]

Answer

(a) $a = 105$ Either $l = 399$ or $d = 7$ $n = 43$ $\rightarrow$ 10 836	B1 B1 B1 B1 [4]	co co co co
(b) $r^2 = 64/144 \rightarrow r = ⅔$ (i) Either $x = ar \rightarrow x = 96$ or $\frac{144}{x} = \frac{x}{64} \rightarrow x = 96$ (ii) Use of $S_\infty = \frac{a}{1-r}$ $\rightarrow$ 432	M1 M1 A1 M1 A1 [5]	award in either part either method ok Used with his *a* and *r* Co (nb do not penalise if *r* and / or *x* negative as well as positive.)

9709/01/O/N/06

Question 9:

Each year a company gives a grant to a charity. The amount given each year increases by 5% of its value in the preceding year. The grant in 2001 was \$5000. Find

(i) the grant given in 2011, [3]

(ii) the total amount of money given to the charity during the years 2001 to 2011 inclusive. [2]

Answer

(i) r = 1.05 with GP 2011 is 11 years. Uses ar^{n-1} $\rightarrow$ \$8 144 (or 8140)	B1 M1 A1 [3]	Anywhere in the question. This could be marked as 2 + 3. Allow if correct formula with $n = 10$ co. (allow 3 sf)
(ii) Use of S_n formula $\rightarrow$ \$ 71 034	M1 A1 [2]	Allow if used correctly with 10 or 11. co (or 71 000)

9709/01/M/J/06

Question 10:

The 1st term of an arithmetic progression is a and the common difference is d, where $d \neq 0$.

(i) Write down expressions, in terms of a and d, for the 5th term and the 15th term. [1]

The 1st term, the 5th term and the 15th term of the arithmetic progression are the first three terms of a geometric progression.

(ii) Show that $3a = 8d$. [3]

(iii) Find the common ratio of the geometric progression. [2]

Answer

. (i) $a+4d$ and $a+14d$	B1 [1]	Both correct.
(ii) $a+4d = ar$, $a+14d = ar^2$ or $\dfrac{a}{a+4d} = \dfrac{a+4d}{a+14d}$ or "$ac=b^2$"	M1	Correct first step – award the mark for both of these starts.
$\rightarrow$ $3a = 8d$	M1 A1 [3]	Correct elimination of r. co. nb answer was given.
(iii) $r = \dfrac{a+4d}{a}$ or $\dfrac{a+14d}{a+4d}$ = 2.5	M1 A1 [2]	Statement + some substitution. co.

9709/01/O/N/07

Question 11:

The second term of a geometric progression is 3 and the sum to infinity is 12.

(i) Find the first term of the progression. [4]

An arithmetic progression has the same first and second terms as the geometric progression.

(ii) Find the sum of the first 20 terms of the arithmetic progression. [3]

Answer

(i)	$ar=3$ and $\dfrac{a}{1-r}=12$	B1 B1	co for each one.
	Solution of sim eqns $\rightarrow$ $a=6$	M1 A1 [4]	Needs to eliminate a or r correctly. co (M mark needs a quadratic)
(ii)	$a=6$, $d=-3$	B1√	For $d = 3 -$ his "6".
	$S_{20} = 10(12 - 57)$	M1	Sum formula must be correct and used.
	$\rightarrow$ -450	A1 [3]	co.

9709/01/M/J/07

Question 12:

The first term of a geometric progression is 81 and the fourth term is 24. Find

(i) the common ratio of the progression, [2]

(ii) the sum to infinity of the progression. [2]

The second and third terms of this geometric progression are the first and fourth terms respectively of an arithmetic progression.

(iii) Find the sum of the first ten terms of the arithmetic progression. [3]

Answer

(i) $a = 81,\ ar^3 = 24$ $\rightarrow r^3 = 24/81 \quad \rightarrow r = ⅔$ or 0.667	M1 A1 [2]	Valid method for r. co
(ii) $S_\infty = \frac{a}{1-r} = 81 \div ⅓ = 243$	M1 A1√ [2]	Correct formula. √ for his a and r, providing $-1 < r < 1$.
(iii) 2nd term of GP $= ar = 81 \times ⅔ = 54$ 3rd term of GP $= ar^2 = 36$ $\rightarrow 3d = -18 \quad (d = -6)$ $\rightarrow S_{10} = 5 \times (108 - 54) = 270$	 M1 M1 A1 [3]	 Finding the 2nd and 3rd terms of GP. M for finding d + correct S_{10} formula. co

9709/01/M/J/08

Question 13:

The first term of an arithmetic progression is 6 and the fifth term is 12. The progression has n terms and the sum of all the terms is 90. Find the value of n. [4]

Answer

3 1^{st} term $= a = 6$ 5^{th} term $= a + 4d = 12$ $\rightarrow d = 1.5$ $S_n = \frac{n}{2}(12 + (n-1)1.5) = 90$ $\rightarrow n^2 + 7n - 120 = 0$ $\rightarrow n = 8$	 B1 M1 DM1 A1 [4]	 Correct value of d Use of correct formula with his d Correct method for soln of quadratic Co (ignore inclusion of $n = -15$)

9709/01/O/N/08

Question 14:

A progression has a second term of 96 and a fourth term of 54. Find the first term of the progression in each of the following cases:

(i) the progression is arithmetic, [3]

(ii) the progression is geometric with a positive common ratio. [3]

Answer

(i) $a + d = 96$ and $a + 3d = 54$ $\rightarrow d = -21 \quad a = 117$	B1 M1A1 [3]	For both expressions. Correct method of solution. co (nb no working, d correct, a wrong 0/3)
(ii) $ar = 96$ and $ar^3 = 54$ $\rightarrow r^2 = \frac{54}{96} \rightarrow r = \frac{3}{4}$ $\rightarrow a = 128$	B1 M1 A1 [3]	For both expressions. Correct method of solution. co. $r = \pm¾$, no penalty.

9709/12/O/N/09

Question 15:

The first term of an arithmetic progression is 8 and the common difference is d, where $d \neq 0$. The first term, the fifth term and the eighth term of this arithmetic progression are the first term, the second term and the third term, respectively, of a geometric progression whose common ratio is r.

(i) Write down two equations connecting d and r. Hence show that $r = \frac{3}{4}$ and find the value of d. [6]

(ii) Find the sum to infinity of the geometric progression. [2]

(iii) Find the sum of the first 8 terms of the arithmetic progression. [2]

Answer

(i) $8+4d=8r$ $8+7d=8r^2$ Eliminates one of the variables $\rightarrow 4r^2-7r+3=0$ Solution $\rightarrow r=\frac{3}{4} \rightarrow d=-\frac{1}{2}$	B1 B1 M1 DM1 A1 A1 [6]	co – but allow if a in place of 8. co – but allow if a in place of 8. Complete elimination of either r or d. Correct method of solution. nb answer for r given. co (assumes $r=\frac{3}{4}$, give B1B1 for equations, B1 for d)
(ii) $S_\infty=\dfrac{a}{1-r} \rightarrow 32$	M1 A1 [2]	Correct formula used.
(iii) $S_8=4(16+7d)$ $=50$	M1 A1 [2]	Correct formula used. $64+28d$ ok co

9709/11/O/N/09

Question 16:

(a) Find the sum to infinity of the geometric progression with first three terms 0.5, 0.5^3 and 0.5^5. [3]

(b) The first two terms in an arithmetic progression are 5 and 9. The last term in the progression is the only term which is greater than 200. Find the sum of all the terms in the progression. [4]

Answer

(a) $a=0.5,\ r=0.5^2$ Uses correct formula $=0.5\div 0.75$ $\rightarrow S_\infty=\frac{2}{3}$ (or 0.667)	B1 M1 A1 [3]	For both a and r. Uses correct formula with some a, r. co.
(b) $a=5,\ d=4$ Uses $200=a+(n-1)d$ or T.I. 50 terms in the progression Use of correct Sum formula $\rightarrow 5150$	M1 A1 M1 A1 [4]	Attempt at finding the number of terms. co. Correct formula (could use the last term (201)). co.

9709/01/M/J/09

Question 17:

(a) The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference. [4]

(b) The first term of a geometric progression is 16 and the fourth term is $\frac{27}{4}$. Find the sum to infinity of the progression. [3]

Answer

(a) $a+4d=18$ $\frac{5}{2}(2a+4d)=75$ Solution $\rightarrow a=12,\ d=1\frac{1}{2}$	B1 B1 M1 A1	[4]	co or $75=5/2(a+18) \rightarrow a=12$ etc co Solution of sim equations co for both
(b) $a=16$ and $ar^3=\dfrac{27}{4}$ $r=\dfrac{3}{4}$ Sum to infinity $=64$	B1 M1 A1	[3]	Needs both of these Correct formula and $\lvert r\rvert<1$

9709/11/O/N/10

Question 18:

The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49.

(i) Find the first term of the progression and the common difference. [4]

The nth term of the progression is 46.

(ii) Find the value of n. [2]

Answer

9^{th} term = 22, $S_4 = 49$ **(i)** $a + 8d = 22$ $2(2a + 3d) = 49$ Soln of sim eqns $\rightarrow d = 1.5, a = 10$	B1 B1 M1 A1 [4]	co co Solution of two linear sim eqns. co
(ii) $a + (n-1)d = 46$ Substitutes for a and d $\rightarrow n = 25$	M1 A1 [2]	Correct formula needed and attempt to solve. co

9709/11/M/J/10

Question 19:

(a) Find the sum of all the multiples of 5 between 100 and 300 inclusive. [3]

(b) A geometric progression has a common ratio of $-\frac{2}{3}$ and the sum of the first 3 terms is 35. Find

(i) the first term of the progression, [3]

(ii) the sum to infinity. [2]

Answer

(a) $a = 100,\ d = 5,$ $n = 41$ $\rightarrow S = 8200$	B1 M1 A1 [3]	co Use of correct sum formula. co
(b) (i) $a + ar + ar^2$ or $a\frac{(1-r^3)}{1-r}$ $= 35 \rightarrow a = 45$	B1 M1 A1 [3]	co Solution of equation. co
(ii) $S_\infty = \frac{a}{1-r} = 27$	M1 A1√ [2]	Correct use of formula. √ for his a.

9709/12/M/J/10

Question 20:

The first term of a geometric progression is 12 and the second term is −6. Find

(i) the tenth term of the progression, [3]

(ii) the sum to infinity. [2]

Answer

(i) $a = 12, ar = -6 \rightarrow r = -½$ $ar^9 = \frac{-3}{128}$	M1 M1 A1 [3]	Attempt at r from "ar" ar^9 must be correct. co
(ii) $S_\infty = \frac{a}{1-r}$ used $\rightarrow 8$	M1 A1 [2]	Correct formula used. M1 needs $\lvert r\rvert < 1$

9709/13/M/J/10

Question 21:

(a) A geometric progression has first term 100 and sum to infinity 2000. Find the second term. [3]

(b) An arithmetic progression has third term 90 and fifth term 80.

(i) Find the first term and the common difference. [2]

(ii) Find the value of m given that the sum of the first m terms is equal to the sum of the first $(m+1)$ terms. [2]

(iii) Find the value of n given that the sum of the first n terms is zero. [2]

Answer

(a) $\frac{100}{1-r}=2000$	M1		Correct formula and attempt to solve
$r=19/20$	A1		
$ar=95$	A1√	[3]	For $100\times r$
(b) (i) $a+2d=90, a+4d=80$			
$d=-5, a=100$	B1B1	[2]	
(ii) $a+md=0$	M1		Or use correct sum formula
$m=20$	A1	[2]	$m=20$ with no working scores 2
(iii) $\frac{n}{2}[200+(n-1)(-5)]=0$	M1		
$n=41$	A1	[2]	$n=41$ with no working scores 2 Do not penalise $n=0$

9709/13/M/J/10

Question 22:

(a) The first and second terms of an arithmetic progression are 161 and 154 respectively. The sum of the first m terms is zero. Find the value of m. [3]

(b) A geometric progression, in which all the terms are positive, has common ratio r. The sum of the first n terms is less than 90% of the sum to infinity. Show that $r^n > 0.1$. [3]

Answer

(a) $d=-7$ used	B1		co
$(m/2)[322+(m-1)(-7)]=0$	M1		Condone omission of $(m/2)$. Statement
47	A1	[3]	co (condone $m=0$)
(b) $\frac{a(1-r^n)}{1-r}<\frac{0.9a}{1-r}$	M1		Allow for $=, <, >, \leq, \geq$
$1-r^n<0.9$	M1		Needs inequality sign correct
$r^n>0.1$	A1	[3]	**co**

9709/12/O/N/10

Question 23:

(a) The sixth term of an arithmetic progression is 23 and the sum of the first ten terms is 200. Find the seventh term. [4]

(b) A geometric progression has first term 1 and common ratio r. A second geometric progression has first term 4 and common ratio $\frac{1}{4}r$. The two progressions have the same sum to infinity, S. Find the values of r and S. [3]

Answer

(a) $a+5d=23$	B1		Solution of 2 linear equations
$5(2a+9d)=200$	B1		
Attempt solution, expect $d=6$ $a=-7$	M1		
29	A1	[4]	

(b) $\frac{1}{1-r}(=)\frac{4}{1-\frac{1}{4}r}$	M1		Use of S_∞ formula twice
$r=\frac{4}{5}$ oe $S=5$	A1A1	[3]	

9709/11/O/N/11

Question 24:

(a) A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. [4]

(b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. [4]

Answer

(a) $ar^2 = 20$	B1	co
$\frac{a}{1-r} = 3a$	B1	co
Soln of equations $\rightarrow$ ($r = ⅔$) $a = 45$	M1 A1 [4]	Complete method to find a. co
(b) $a + 7d = 3(a + 2d)$	M1	Use of $a + (n-1)d$
$\rightarrow 2a = d$	A1	co
$S_8 = 4(2a + 7d) = 32d$ or $64a$	M1	correct use of S_n formula once.
$S_4 = 2(2a + 3d) = 8d$ or $16a$	A1 [4]	ag

9709/13/M/J/11

Question 25:

(a) A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector. [6]

(b) The first, second and third terms of a geometric progression are $2k + 3$, $k + 6$ and k, respectively. Given that all the terms of the geometric progression are positive, calculate

(i) the value of the constant k, [3]

(ii) the sum to infinity of the progression. [2]

Answer

(a) $a + 5d = 4a$ or $\frac{(a+4a)}{2} \times 6$	B1	co
$\frac{6}{2}(2a + 5d)$ or $\frac{(a+4a)}{2} \times 6 = 360$	M1 A1	Correct left-hand side. All correct.
Sim Eqns $a = 24°$ or $\frac{2\pi}{15}$ rads	A1	Either answer.
Arc length = $\frac{50}{}$	M1	Correct use of arc length with θ in rads.
Perimeter = 12.1.	A1 [6]	co
(b) (i) $\frac{k+6}{2k+3} = \frac{k}{k+6}$	M1 A1	Correct eqn for k.
$\rightarrow k^2 - 9k - 36 = 0 \rightarrow k = 12$ (NB stating a, ar, ar^2 as f(k) gets M1)	A1 [3]	Co condone inclusion of $k = -3$.
(ii) $r = ⅔$, $a = 27$ $\rightarrow S_\infty = 27 \div ⅓ = 81$.	M1 A1 [2]	Correct formula for S_∞ must have $-1 \le r \le 1$. co

9709/12/M/J/11

Question 26:

A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

(i) if Model 1 is used, [4]

(ii) if Model 2 is used. [3]

Answer

(i) 1000, 2000, 3000... or 50, 100, 150...	M1	Recognise series, correct *a*/*d* (or 3 terms)
$\frac{40}{2(1000+40000)}$ or $\frac{40}{2(2000+39000)}$	M1	Correct use of formula
× 5% of attempt at valid sum	M1	Can be awarded in either **(i)** or **(ii)**
41000	A1 [4]	cao
(ii) 1000, 1000 × 1.1, 1000 × 1.1^2 + ... or with $a = 50$	M1	Recognise series, correct *a*/*r* (or 3 terms)
$\frac{1000(1.1^{40}-1)}{1.1-1}$	M1	Correct use of formula. Allow e.g. $r = 0.1$
22100	A1 [3]	Or answers rounding to this

9709/11/M/J/11

Question 27:

(a) An arithmetic progression contains 25 terms and the first term is −15. The sum of all the terms in the progression is 525. Calculate

(i) the common difference of the progression, [2]

(ii) the last term in the progression, [2]

(iii) the sum of all the positive terms in the progression. [2]

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be $4000 in 2012 and will increase by 5% each year. Calculate

(i) the value of the grant in 2022, [2]

(ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]

Answer

(a) $a=-15$, $n=25$		
(i) Use of S_n → $d = 3$.	M1 A1 [2]	Must be correct formula. co
(ii) Last term = $a + 24d$ → 57 (or $525 = ½ × 25 × (−15 + l) → l = 57$)	M1 A1√ [2]	Must be $a + 24d$ √ for his *d*.
(iii) Positive terms are 3,6, ...57 Either $a = 0$ or 3, $n = 19$ or 20 Use of S_{19} or S_{20} → 570	M1 A1 [2]	Correct use of formula for S_n. co
(b) $r = 1.05$	B1	In either part **(i)** or **(ii)**.
(i) 11th term = ar^{10} = $6516 or $6520	B1 [2]	co
(ii) $S_{11} = \frac{4000\times(1.05^{11}-1)}{.05}$ = $56800 or (56827)	M1 A1 [2]	Correct sum formula with their *r*. co

9709/12/O/N/11

Question 28:

The first and second terms of a progression are 4 and 8 respectively. Find the sum of the first 10 terms given that the progression is

(i) an arithmetic progression, [2]

(ii) a geometric progression. [2]

Answer

(i) $5[8+9\times 4]$ 220	M1 A1 [2]	Use correct formula with a=4, d=4
(ii) $\frac{4(2^{10}-1)}{2-1}$ 4092	M1 A1 [2]	Use correct formula with a=4, r=2 or ½ 4090 without 4092 A0

9709/13/O/N/11

Question 29:

(a) The first two terms of an arithmetic progression are 1 and $\cos^2 x$ respectively. Show that the sum of the first ten terms can be expressed in the form $a - b\sin^2 x$, where a and b are constants to be found. [3]

(b) The first two terms of a geometric progression are 1 and $\frac{1}{3}\tan^2\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(i) Find the set of values of θ for which the progression is convergent. [2]

(ii) Find the exact value of the sum to infinity when $\theta = \frac{1}{6}\pi$. [2]

Answer

(a) $S_{10} = \frac{10}{2}[2+9(\cos^2 x - 1)]$	**M1**	Correct formula with $d = \pm(\cos^2 x - 1)$
$S_{10} = 5[2 - 9\sin^2 x]$	**M1**	Use of $c^2 + s^2 = 1$ in a correct S_{10}
$S_{10} = 10 - 45\sin^2 x$	**A1** **[3]**	Or $a = 10$, $b = 45$
(b) (i) $(0<)\frac{1}{3}\tan^2\theta < 1$ oe	**M1**	Allow $\leqslant$
$(0<)\theta < \frac{\pi}{3}$	**A1** **[2]**	cao Allow $\leqslant$
(ii) $S_\infty = \frac{1}{1-\frac{1}{3}\tan^2\frac{\pi}{6}}$	**M1**	
$S_\infty = \frac{9}{8}$ or 1.125	**A1** **[2]**	cao

9709/11/M/J/12

Question 30:

(a) In an arithmetic progression, the sum of the first n terms, denoted by S_n, is given by

$$S_n = n^2 + 8n.$$

Find the first term and the common difference. [3]

(b) In a geometric progression, the second term is 9 less than the first term. The sum of the second and third terms is 30. Given that all the terms of the progression are positive, find the first term. [5]

Answer

(a) $S_n = n^2 + 8n$.		
$S_1 = 9 \rightarrow a = 9$ $S_2 = 20 \rightarrow a + d = 11 \rightarrow d = 2$ (or equating $n^2 + 8n$ with S_n and comparing coefficients)	B1 M1 A1 [3]	co Realises that S_2 is $a + (a + d)$. co

(b) $a - ar = 9$ $ar + ar^2 = 30$ Eliminates a $\rightarrow$ $3r^2 + 13r - 10 = 0$ or $\rightarrow$ $2a^2 - 57a + 81 = 0$ $\rightarrow$ $r = ⅔$ $\rightarrow$ $a = 27$	B1 B1 M1 A1 A1 [5]	co co Complete elimination of r or a Correct quadratic. co (condone 27 or 1.5)

9709/12/M/J/12

Question 31:

The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

(i) Find the common difference of the progression. [2]

The first term, the ninth term and the nth term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

(ii) Find the common ratio of the geometric progression and the value of n. [5]

Answer

(i) Uses S_n $\frac{9}{2}(24 + 8d) = 135 \rightarrow d = ¾$	M1 A1 [2]	Uses correct formula co
(ii) 9^{th} term of AP = $12 + 8 \times ¾ = 18$ GP 1^{st} tern 12, 2^{nd} term 18 Common ratio = $r = 18 \div 12 = 1½$ 3^{rd} term of GP = $ar^2 = 27$ nth term of AP is $12 + (n - 1)¾$ $12 + (n - 1)¾ = 27 \rightarrow n = 21$	B1√ M1 M1 M1A1 [5]	√ on "d" Uses "ar" Uses ar^2 or "ar" × r Links AP with GP. co

9709/13/M/J/12

Question 32:

The first term of a geometric progression is $5\frac{1}{3}$ and the fourth term is $2\frac{1}{4}$. Find

(i) the common ratio, [3]

(ii) the sum to infinity. [2]

Answer

(i) $2\frac{1}{4} = 5\frac{1}{3}r^3$ $r^3 = \frac{9}{4} \times \frac{3}{16} = \frac{27}{64}$ $r = \frac{3}{4}$ or 0.75	M1 A1 A1 [3]	
(ii) $S_\infty = \frac{5\frac{1}{3}}{1 - \frac{3}{4}} = \frac{64}{3}$ (or $21\frac{1}{3}$ or 21.3)	M1 A1 [2]	cao

9709/13/O/N/12

Question 33:

The first term of an arithmetic progression is 61 and the second term is 57. The sum of the first n terms is n. Find the value of the positive integer n. [4]

Answer

$\frac{n}{2}[122 + (n-1)(-4)]$ $n = \frac{n}{2}[122 + (n-1)(-4)]$ $2n(n - 31) = 0$ $n = 31$	**M1** **A1** **DM1** **A1** **[4]**	Attempt sum formula with $a = 61$, $d = -4$ Equated to n cao Attempt to solve. Accept div. by n cao

9709/11/O/N/12

Question 34:

(a) In a geometric progression, all the terms are positive, the second term is 24 and the fourth term is $13\frac{1}{2}$. Find

(i) the first term, [3]

(ii) the sum to infinity of the progression. [2]

(b) A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are 3° and 5°. Find the value of n. [4]

Answer

(a) (i) $ar = 24,\ ar^3 = 13\tfrac{1}{2}$		
Eliminates a (or r) $\rightarrow r = \tfrac{3}{4}$	B1	Both needed
$\rightarrow a = 32$	M1	Method of Solution.
	A1	co
	[3]	
(ii) sum to infinity $= 32 \div \tfrac{1}{4} = 128$		
	M1A1√	Correct formula used. √ on value of r
	[2]	
(b) $a = 3,\ d = 2$	B1	Correct value for d
$\frac{n}{2}(6+(n-1)2)\quad (= 360)$	M1	Correct S_n used. no need for 360 here.
$\rightarrow 2n^2 + 4n - 720 = 0$	A1	Correct quadratic
$\rightarrow n = 18$	A1	co
	[4]	

9709/12/O/N/12

Question 35:

(a) In an arithmetic progression the sum of the first ten terms is 400 and the sum of the next ten terms is 1000. Find the common difference and the first term. [5]

(b) A geometric progression has first term a, common ratio r and sum to infinity 6. A second geometric progression has first term $2a$, common ratio r^2 and sum to infinity 7. Find the values of a and r. [5]

Answer

(a) $\frac{10}{2}(2a+9d) = 400$ oe	B1	$\rightarrow 2a+9d = 80$
$\frac{20}{2}(2a+19d) = 1400$ OR		
$\frac{10}{2}[2(a+10d)+9d] = 1000$	B1	$\rightarrow 2a+19d = 140$ or $2a+29d = 200$
$d = 6 \quad a = 13$	M1A1A1 [5]	Solve sim. eqns both from S_n formulae
(b) $\frac{a}{1-r} = 6 \qquad \frac{2a}{1-r^2} = 7$	B1B1	
$\frac{12(1-r)}{1-r^2} = 7$ or $\frac{1-r^2}{1-r} = \frac{12}{7}$	M1	Substitute or divide
$r = \frac{5}{7}$ or 0.714	A1	
$a = \frac{12}{7}$ or 1.71(4)	A1√ [5]	Ignore any other solns for r and a

9709/11/O/N/13

Question 36:

The third term of a geometric progression is -108 and the sixth term is 32. Find

(i) the common ratio, [3]

(ii) the first term, [1]

(iii) the sum to infinity. [2]

Answer

(i) $ar^2 = -108,\ ar^5 = 32$	B1	
$r^3 = \dfrac{32}{-108} = \left(-\dfrac{8}{27}\right)$	M1	Eliminating a
$r = \left(-\dfrac{2}{3}\right)$ or -0.666 or -0.667	A1 [3]	$-\dfrac{2}{3}$ from little or no working $\rightarrow \dfrac{3}{3}$ www
(ii) $a = -243$	B1 $\sqrt{}$ [1]	ft on *their* r $\left(-\dfrac{108}{r^2} \text{ or } \dfrac{32}{r^5}\right)$
(iii) $S_\infty = \dfrac{-243}{1+\dfrac{2}{3}} = -\dfrac{729}{5}$ or -145.8	M1A1 [2]	Accept -146. For M1 $\lvert r \rvert$ must be < 1

9709/11/M/J/13

Question 37:

(a) The first and last terms of an arithmetic progression are 12 and 48 respectively. The sum of the first four terms is 57. Find the number of terms in the progression. [4]

(b) The third term of a geometric progression is four times the first term. The sum of the first six terms is k times the first term. Find the possible values of k. [4]

Answer

(a) $57 = 2(24 + 3d) \rightarrow d = 1.5$		
$48 = 12 + (n-1)1.5 \rightarrow n = 25$	M1 A1	Use of correct S_n formula.
	M1 A1 [4]	Use of correct T_n formula.
(b) $ar^2 = 4a \quad r = \pm 2$	B1	(allow for $r = 2$)
$\dfrac{a(r^6-1)}{r-1} = ka$	B1	
$\rightarrow k = 63$ or $k = -21$	B1 B1 [4]	

9709/12/M/J/13

Question 38:

(a) In a geometric progression, the sum to infinity is equal to eight times the first term. Find the common ratio. [2]

(b) In an arithmetic progression, the fifth term is 197 and the sum of the first ten terms is 2040. Find the common difference. [4]

Answer

(a) $\dfrac{a}{1-r} = 8a \Rightarrow 1(a) = 8(a)(1-r)$	B1	
$r = \dfrac{7}{8}$ oe	B1 [2]	
(b) $a + 4d = 197$	B1	Or $2a + 9d = 408$
$\dfrac{10}{2}[2a + 9d] = 2040$	B1	Attempt to solve simultaneously
$d = 14$	M1A1 [4]	

9709/13/O/N/13

Question 39:

(a) In an arithmetic progression, the sum, S_n, of the first n terms is given by $S_n = 2n^2 + 8n$. Find the first term and the common difference of the progression. [3]

(b) The first 2 terms of a geometric progression are 64 and 48 respectively. The first 3 terms of the geometric progression are also the 1st term, the 9th term and the nth term respectively of an arithmetic progression. Find the value of n. [5]

Answer

(a)	$S_n = 2n^2 + 8n$ $S_1 = 10 \;\; = a$ $S_2 = 24 = a + (a + d)\; d = 4$	 B1 M1 A1	 [3]	 correct use of S_n formula.
(b)	GP $a = 64\; ar = 48 \rightarrow r = \frac{3}{4}$ $\rightarrow$3rd term is $ar^2 = 36$ AP $a = 64,\;\; a + 8d = 48 \;\rightarrow\; d = -2$ $36 = 64 + (n-1)(-2)$ $\rightarrow n = 15.$	B1 M1 B1 M1 A1	 [5]	 ar^2 numerical – for their r correct use of $a+(n-1)d$

9709/13/M/J/13

Question 40:

(a) An athlete runs the first mile of a marathon in 5 minutes. His speed reduces in such a way that each mile takes 12 seconds longer than the preceding mile.

(i) Given that the nth mile takes 9 minutes, find the value of n. [2]

(ii) Assuming that the length of the marathon is 26 miles, find the total time, in hours and minutes, to complete the marathon. [2]

(b) The second and third terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [4]

Answer

(a) (i) $a = 300, d = 12$ $\rightarrow 540 = 300 + (n-1)12 \rightarrow n = 21$ (ii) $S_{26} = 13\,(600 + 25{\times}12) = 11700$ $\rightarrow$ 3 hours 15 minutes.	 M1 A1 [2] M1 A1 [2]	 Use of nth term. Ans 20 gets 0. Ignore incorrect units Correct use of s_n formula.
(b) $ar = 48$ and $ar^2 = 32 \rightarrow r = \frac{2}{3}$ $\rightarrow a = 72.$ $S_\infty = 72 \div \frac{1}{3} = 216.$	M1 A1 M1 A1√ [4]	Needs ar and ar^2 + attempt at a and r. Correct S_∞ formula with $\lvert r \rvert < 1$

9709/12/O/N/13

Question 41:

(a) The sum, S_n, of the first n terms of an arithmetic progression is given by $S_n = 32n - n^2$. Find the first term and the common difference. [3]

(b) A geometric progression in which all the terms are positive has sum to infinity 20. The sum of the first two terms is 12.8. Find the first term of the progression. [5]

Answer

(a) $S_n = 32n - n^2$. Set n to 1, a or $S_1 = 31$ Set n to 2 or other value $S_2 = 60$ $\rightarrow$ 2nd term = 29 $\rightarrow$ $d = -2$ (or equates formulae – compares coeffs n^2, n) [M1 comparing, A1 d A1 a]	 B1 M1 A1 [3]	 co Correct method. co [M1 only when coeffs compared]

(b)	$\frac{a}{1-r}=20$, $\frac{a(1-r)^2}{1-r}$, or $a+ar=12.8$	B1 B1	co co
	Elimination of $\frac{a}{1-r}$ or a or r	M1	'Correct' elimination to form equation in a or r
	$\rightarrow$ $(r=0.6)$ $\rightarrow$ $a=8$	DM1 A1 [5]	Complete method leading to $a=$ Condone $a=8$ and 32

9709/12/O/N/14

Question 42:

(i) A geometric progression has first term a $(a \neq 0)$, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio $2r$ and sum to infinity $3S$. Find the value of r. [3]

(ii) An arithmetic progression has first term 7. The nth term is 84 and the $(3n)$th term is 245. Find the value of n. [4]

Answer

(i)	$S=\frac{a}{1-r}$, $3S=\frac{a}{1-2r}$	**B1**	At least $3S=\frac{a}{1-2r}$
	$1-r=3-6r$	**M1**	Eliminate S
	$r=\frac{2}{5}$	**A1** **[3]**	
(ii)	$7+(n-1)d=84$ and/or $7+(3n-1)d=245$	**B1**	At least one of these equations seen
	$[(n-1)d=77,\ (3n-1)d=238,\ 2nd=161]$	**B1**	Two different seen – unsimplified ok
	$\frac{n-1}{3n-1}=\frac{77}{238}$ (must be from the correct u_n formula)	**M1**	Or other attempt to elim d. E.g. sub $d=\frac{161}{2n}$ (if n is eliminated d must be found)
	$n=23$ $(d=\frac{77}{22}=3.5)$	**A1** **[4]**	

9709/11/O/N/14

Question 43:

The 1st, 2nd and 3rd terms of a geometric progression are the 1st, 9th and 21st terms respectively of an arithmetic progression. The 1st term of each progression is 8 and the common ratio of the geometric progression is r, where $r \neq 1$. Find

(i) the value of r, [4]

(ii) the 4th term of each progression. [3]

Answer

(i)	GP 8 $8r$ $8r^2$ AP 8 $8+8d$ $8+20d$		
	$8r=8+8d$ and $8r^2=8+20d$	B1 B1	B1 for each equation.
	Eliminates d $\rightarrow$ $2r^2-5r+3=0$	M1	Correct elimination.
	$\rightarrow$ $r=1.5$ (or 1)	A1 [4]	co (no penalty for including $r=1$)
(ii)	4th term of GP $=ar^3=8\times 27/8=27$	B1√	co
	If $r=1.5$, $d=0.5$ 4th term of AP $=a+3d=9½$	M1A1 [3]	needs $a+3d$ and correct method for d

9709/12/M/J/14

Question 44:

Three geometric progressions, P, Q and R, are such that their sums to infinity are the first three terms respectively of an arithmetic progression.

Progression P is $2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \ldots$.

Progression Q is $3,\ 1,\ \frac{1}{3},\ \frac{1}{9},\ \ldots$.

(i) Find the sum to infinity of progression R. [3]

(ii) Given that the first term of R is 4, find the sum of the first three terms of R. [3]

Answer

(i) $S_P = \dfrac{2}{1-\frac{1}{2}}, S_P = \dfrac{3}{1-\frac{1}{3}}$	**M1**	At least one correct
$S_P = 4, S_Q = \dfrac{9}{2}$	**A1**	At least one correct
$S_R = 5$ cao	**A1** **[3]**	
(ii) $\dfrac{4}{1-r} = their\ S_R$	**M1**	
$r = \dfrac{1}{5}$	**A1**	
$R = 4 + \dfrac{4}{5} + \dfrac{4}{25} = 4\dfrac{24}{25}$ or 4.96 cao	**A1** **[3]**	

9709/13/O/N/14

Question 45:

An arithmetic progression has first term a and common difference d. It is given that the sum of the first 200 terms is 4 times the sum of the first 100 terms.

(i) Find d in terms of a. [3]

(ii) Find the 100th term in terms of a. [2]

Answer

(i) $200/2(2a + 199d) = 4 \times 100/2(2a + 99d)$	M1A1	Correct formula used (once) M1, correct eqn A1
$d = 2a$ cao	A1 [3]	
(ii) $a + 99d = a + 99 \times 2a$	M1	Sub. *their* part(i) into correct formula
$199a$ cao	A1 [2]	

9709/11/M/J/14

Question 46:

The first term in a progression is 36 and the second term is 32.

(i) Given that the progression is geometric, find the sum to infinity. [2]

(ii) Given instead that the progression is arithmetic, find the number of terms in the progression if the sum of all the terms is 0. [3]

Answer

36, 32, …		
(i) $r = \frac{8}{9}$ $S_\infty = (their\ a) \div (1 - their\ r)$	M1	Method for r and S_∞ ok. ($\lvert r \rvert < 1$)
$S_\infty = 36 \div \frac{1}{9} = 324$	A1 [2]	co
(ii) $d = -4$	B1	co
$0 = \frac{n}{2}(72 + (n-1)(-4))$	M1	S_n formula ok and a value for d $\left(\neq \frac{8}{9}\right)$
$\rightarrow n = 19$	A1 [3]	Condone $n = 0$ but no other soln

9709/13/M/J/14

Question 47:

The first term of a progression is $4x$ and the second term is x^2.

(i) For the case where the progression is arithmetic with a common difference of 12, find the possible values of x and the corresponding values of the third term. [4]

(ii) For the case where the progression is geometric with a sum to infinity of 8, find the third term. [4]

Answer

(i)	$x^2 - 4x = 12$ $x = -2$ or 6 3^{rd} term $= (-2)^2 + 12 = 16$ or $6^2 + 12 = 48$	M1 A1 A1A1 [4]	$4x - x^2 = 12$ scores M1A0 SC1 for 16, 48 after $x = 2, -6$
(ii)	$r^2 = \frac{x^2}{4x}\left(= \frac{x}{4}\right)$ soi	M1	
	$\frac{4x}{1-\frac{x}{4}} = 8$	M1	Accept use of unsimplified $\frac{x^2}{4x}$ or $\frac{4x}{x^2}$ or $\frac{4}{x}$
	$x = \frac{4}{3}$ or $r = \frac{1}{3}$	A1	
	3^{rd} term $= \frac{16}{27}$ (or 0.593)	A1 [4]	
	ALT $\frac{4x}{1-r} = 8 \to r = 1 - \frac{1}{2}x$ **or** $\frac{4x}{1-r} = 8 \to x = 2(1-r)$	M1	
	$x^2 = 4x\left(1 - \frac{1}{2}x\right)$ $\quad r = \frac{2(1-r)}{4}$	M1	
	$x = \frac{4}{3}$ $\quad r = \frac{1}{3}$	A1	

9709/11/O/N/15

Question 48:

(a) The third and fourth terms of a geometric progression are $\frac{1}{3}$ and $\frac{2}{9}$ respectively. Find the sum to infinity of the progression. [4]

(b) A circle is divided into 5 sectors in such a way that the angles of the sectors are in arithmetic progression. Given that the angle of the largest sector is 4 times the angle of the smallest sector, find the angle of the largest sector. [4]

Answer

(a)	$ar^2 = \frac{1}{3}$, $ar^3 = \frac{2}{9}$ $\to r = \frac{2}{3}$ aef	M1	Any valid method, seen or implied. Could be answers only.
	Substituting $\to a = \frac{3}{4}$	A1	Both a and r
	$\to S_\infty = \frac{\frac{3}{4}}{\frac{1}{3}} = 2\frac{1}{4}$ aef	M1 A1 [4]	Correct formula with $\lvert r \rvert < 1$, cao
(b)	$4a = a + 4d \quad \to 3a = 4d$	B1	May be implied in $360 = 5/2(a + 4a)$
	$360 = S_5 = \frac{5}{2}(2a + 4d)$ or $12.5a$	M1	Correct S_n formula or sum of 5 terms
	$\to a = 28.8°$ aef Largest $= a + 4d$ or $4a = 115.2°$ aef	A1 B1 [4]	cao, may be implied (may use degrees or radians)

9709/11/M/J/15

Question 49:

A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

Model A : The height reached is reduced by 0.04 metres each time the ball bounces.

Model B : The height reached is reduced by 4% each time the ball bounces.

(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

(b) using model B. [3]

(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]

Answer

(i)	(a)	$1.92 + 1.84 + 1.76 + \ldots$ oe $\frac{20}{2}[2\times1.92+19\times(-0.08)]$ oe 23.2 cao	B1 M1 A1 [3]	OR a=0.96, d= –.04 & ans doubled/adjusted Corr formula used with corr d & *their* a, n $a=1, n=21 \rightarrow 12.6\ (25.2)$, $a=0.96, n=21 \rightarrow 11.76\ (23.52)$
	(b)	$1.92+1.92(.96)+1.92(.96)^2+\ldots$ $\frac{1.92(1-.96^{20})}{1-.96}$ 26.8 cao	B1 M1 A1 [3]	OR a=.96, r =.96 & ans /doubled/adjusted Corr formula used with r =.96 & *their* a, n $a=.96, n=21 \rightarrow 13.82\ (27.63)$ $a=1, n=21 \rightarrow 14.39\ (28.78)$
	(ii)	$\frac{1.92}{1-.96}=48$ or $\frac{0.96}{1-0.96}=24$ & then Double AG	M1A1 [2]	$a=1 \rightarrow 25\ (50)$ but must be doubled for M1 $1.92\frac{(1-0.96^n)}{1-0.96}<48 \rightarrow 0.96^n>0$ (www) 'which is true' scores SCB1

9709/13/O/N/15

Question 50:

(a) The first term of an arithmetic progression is −2222 and the common difference is 17. Find the value of the first positive term. [3]

(b) The first term of a geometric progression is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0 < \theta < \pi$. Find the set of values of θ for which the progression is convergent. [5]

Answer

(a)	2222/17 (=131 or 130.7) 131 × 17 (=2227) −2222 + 2227 = 5	M1 M1 A1 [3]	Ignore signs. Allow 2239/17→131.7 or 132 Ignore signs. Use 131. 5 www gets 3/3
(b)	$r=\frac{2\cos\theta}{\sqrt{3}}$ soi oe $(-1<)\frac{2\cos\theta}{\sqrt{3}}<1$ or $(0<)\frac{2\cos\theta}{\sqrt{3}}<1$ soi $\pi/6,\ 5\pi/6$ soi (but dep. on M1) $\pi/6<\theta<5\pi/6$ cao	B1 M1√ A1A1 A1 [5]	 Ft on *their r*. Ignore a 2nd inequality on LHS Allow 30°, 150°. Accept ⩽

9709/13/M/J/15

Question 51:

A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

Answer

(i) (a)	$a+(n-1)d = 10+29\times 2$	M1	Use of nth term of an AP with a=±10, d=±2, n=30 or 29
	$= 68$	A1 [2]	Condone $-68 \to 68$
(b)	$\frac{1}{2}n(20+2(n-1)) = 2000$ or 0	M1	Use of S_n formula for an AP with a=±10, d=±2 and equated to either 0 or 2000.
	$\to 2n^2 + 18n - 4000 = 0$ oe	A1	Correct 3 term quadratic = 0.
	(n=) 41	A1 [3]	
(ii)	$r = 1.1$, oe	B1	e.g. $\frac{11}{10}$, 110%
	Uses $S_{30} = \frac{10(1.1^{30}-1)}{1.1-1}$ (= 1645)	M1	Use of S_n formula for a GP, a=±10, n=30.
	Percentage lost $= \frac{2000-1645}{2000}\times 100$	DM1	Fully correct method for % left with "their 1645"
	$= 17.75$	A1 [4]	allow 17.7 or 17.8.

9709/12/M/J/16

Question 52:

The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

Answer

$r = \frac{3+2d}{3}$ or $\frac{3+12d}{3+2d}$ or $r^2 = \frac{3+12d}{3}$	B1	1 correct equation in r and d only is sufficient
$(3+2d)^2 = 3(3+12d)$ oe OR sub $2d = 3r - 3$	M1	Eliminate r or d using valid method
$(4)d(d-6) = 0$ OR $3r^2 = 18r - 15 \to (r-1)(r-5)$	DM1	Attempt to simplify and solve quadratic
$d = 6$ $r = 5$	A1 A1 [5]	Ignore $d = 0$ or $r = 1$ Do not allow -5 or ± 5

9709/13/M/J/16

Question 53:

The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]

Answer

$a(1+r)=50$ or $\dfrac{a(1-r^2)}{1-r}=50$	B1		
$ar(1+r)=30$ or $\dfrac{a(1-r^3)}{1-r}=30+a$	B1		Or otherwise attempt to solve for r
Eliminating a or r	M1		Any correct method
$r=3/5$	A1		
$a=125/4$ oe	A1		
$S=625/8$ oe	A1√	[6]	Ft through on *their* r and a $(-1<r<1)$

9709/11/O/N/16

Question 54:

(a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

Answer

(a) (i)	$200+(15-1)(+/-5)$ $=130$	M1 A1	[2]	Use of *n*th term with $a=200$, $n=14$ or 15and $d=+/-5$.
(ii)	$\frac{n}{2}[400+(n-1)(+/-5)]=(3050)$ $\rightarrow\ 5n^2-405n+6100\ (=0)$ $\rightarrow\ 20$	M1 A1 A1	[3]	Use of S_n a=200 and $d=+/-5$.
(b) (i)	$ar^2, ar^5 \ \rightarrow\ r=½$ $\dfrac{63}{2}=\dfrac{a(1-½^6)}{½}\ \rightarrow\ a=16$	M1 A1 M1 A1	[4]	Both terms correct. Use of $S_n=31.5$ with a numeric r.
(ii)	Sum to infinity $=\dfrac{16}{½}=32$	B1√	[1]	√ for their a and r with $\lvert r\rvert<1$.

9709/12/O/N/16

Question 55:

(a) Two convergent geometric progressions, P and Q, have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos\theta$ and the second term is $\cos\theta+\sin^2\theta$, where $0\leqslant\theta\leqslant\pi$. The sum of the first 13 terms is 52. Find the possible values of θ. [5]

Answer

(a)	$\dfrac{6}{1-r}=\dfrac{12}{1+r}$ $r=\dfrac{1}{3}$ $S=9$	M1 A1 A1	[3]	

9709/13/O/N/16

Question 56:

(a) The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]

(b)	$\frac{13}{2}\left[2\cos\theta+12\sin^2\theta\right]=52$	M1*		Use of correct formula for sum of AP
	$2\cos\theta+12(1-\cos^2\theta)=8 \to 6\cos^2\theta-\cos\theta-2(=0)$	DM1		Use $s^2=1-c^2$ & simplify to 3-term quad
	$\cos\theta=2/3$ or $-1/2$ soi	A1		
	$\theta=0.841$, 2.09 Dep on previous A1	A1A1	[5]	Accept 0.268π, 2π/3. SRA1 for 48.2°, 120° Extra solutions in range −1

(b) The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x+2\cos x)$ respectively, where x is an acute angle.

(i) Show that $\tan x=\frac{4}{3}$. [3]

(ii) Find the sum of the first twenty terms of the progression. [3]

Answer

(a)		$a=50$, $ar^2=32$	B1	seen or implied
		$\to r=\frac{4}{5}$ (allow $-\frac{4}{5}$ for M mark)	M1	Finding r and use of correct S_∞ formula
		$\to S_\infty=250$	A1 [3]	Only if $\|r\|<1$
(b)	(i)	2sin$x$, 3cos$x$, (sin$x$ + 2cos$x$). $3c-2s=(s+2c)-3c$ (or uses a, $a+d$, $a+2d$)	M1	Links terms up with AP, needs one expression for d.
		$\to 4c=3s \to t=\frac{4}{3}$	M1 A1 [3]	Arrives at $t=k$. ag
		SC uses $t=\frac{4}{3}$ to show		
		$u_1=\frac{8}{5},u_2=\frac{9}{5},u_3=\frac{10}{5}$, **B1** only		
	(ii)	$\to c=\frac{3}{5}$, $s=\frac{4}{5}$ or calculator $x=53.1°$	M1	
		$\to a=1.6$, $d=0.2$	M1	Correct method for both a and d.
		$\to S_{20}=70$	A1 [3]	(Uses S_n formula)

9709/11/M/J/16

Question 57:

(a) A geometric progression has first term $3a$ and common ratio r. A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r. [3]

(b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n. [3]

Answer

(i)	$\frac{3a}{1-r}=\frac{a}{1+2r}$	M1	Attempt to equate 2 sums to infinity. At least one correct
	$3+6r=1-r$	DM1	Elimination of 1 variable (a) at any stage and multiplication
	$r=-\frac{2}{7}$	A1	
		3	
(ii)	$½n\left[2\times15+(n-1)4\right]=½n\left[2\times420+(n-1)(-5)\right]$	M1A1	Attempt to equate 2 sum to n terms, at least one correct (M1). Both correct (A1)
	$n=91$	A1	
		3	

9709/11/O/N/17

Question 58:

(a) Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for \$10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest \$100. [2]

(b) The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n+7)$. Find the 1st term and the common difference of the progression. [4]

Answer

Question	Answer	Marks	Guidance
(a)	Uses $r = (1.05 \text{ or } 105\%)^{9,\ 10 \text{ or } 11}$	B1	Used to multiply repeatedly or in any GP formula.
	New value $= 10000 \times 1.05^{10}$ = (\$)16 300	B1	
		2	
(b)	*EITHER:* $n = 1 \to 5 \qquad a = 5$	(B1	Uses $n = 1$ to find a
	$n = 2 \to 13$	B1	Correct S_n for any other value of n (e.g. $n = 2$)
	$a + (a + d) = 13 \quad \to d = 3$	M1 A1)	Correct method leading to d =
	OR: $\left(\frac{n}{2}\right)(2a+(n-1)d) = \left(\frac{n}{2}\right)(3n+7)$		$\left(\frac{n}{2}\right)$ maybe be ignored
	$\therefore dn + 2a - d = 3n + 7 \to dn = 3n \to d = 3$	(*M1A1	Method mark awarded for equating terms in n from correct S_n formula.
	$2a - (\text{their } 3) = 7, \quad a = 5$	DM1 A1)	
		4	

9709/12/O/N/17

Question 59:

An arithmetic progression has first term -12 and common difference 6. The sum of the first n terms exceeds 3000. Calculate the least possible value of n. [4]

Answer

(a)	$(S_n =)\ \frac{n}{2}[32 + (n-1)8]$ and 20000	M1	M1 correct formula used with d from $16 + d = 24$
		A1	A1 for correct expression linked to 20000.
	$\to n^2 + 3n - 5000\ (<, =, > 0)$	DM1	Simplification to a three term quadratic.
	$\to (n = 69.2) \to 70$ terms needed.	A1	Condone use of 20001 throughout. Correct answer from trial and improvement gets 4/4.

9709/13/O/N/17

Question 60:

(a) An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of -28. Find the sum of all the terms in the progression. [4]

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated \$2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

Answer

Question	Answer	Marks	Guidance
(a)	$a = 32,\ a + 4d = 22,\ \rightarrow d = -2.5$	B1	
	$a + (n-1)d = -28 \rightarrow n = 25$	B1	
	$S_{25} = \frac{25}{2}(64 - 2.5 \times 24) = 50$	M1 A1	M1 for correct formula with $n = 24$ or $n = 25$
	Total:	4	
(b)	$a = 2000,\ r = 1.025$	B1	$r = 1 + 2.5\%$ ok if used correctly in S_n formula
	$S_{10} = 2000(\frac{1.025^{10} - 1}{1.025 - 1}) = 22400$ or a value which rounds to this	M1 A1	M1 for correct formula with $n = 9$ or $n = 10$ and their a and r
			SR: correct answer only for $n = 10$ **B3**, for $n = 9$, **B1** (£19 900)
	Total:	3	

9709/11/M/J/17

Question 61:

(a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

Answer:

(b)	$a = 6,\ \frac{a}{1-r} = 18 \rightarrow r = ⅔$	M1A1	Correct $S\infty$ formula used to find r.
	New progression $a = 36,\ r = \frac{4}{9}$ oe	M1	Obtain new values for a and r by any valid method.
	New $S\infty = \frac{36}{1-\frac{4}{9}} \rightarrow 64.8$ or $\frac{324}{5}$ oe	A1	(Be aware that $r = -⅔$ leads to 64.8 but can only score M marks)
	Total:	4	

9709/12/M/J/17

Question 62:

The common ratio of a geometric progression is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$. [2]

(ii) Find the set of possible values that S can take.

Answer:

(ii)	Single range $1 < S < 3$ or (1, 3)	B2	Accept $1 < 2 - r < 3$. Correct range but with $S = 2$ omitted scores SR **B1** $1 \leqslant S \leqslant 3$ scores SR **B1**. [$S > 1$ and $S < 3$] scores SR **B1**.
	$1-r$ $1-r$ $\frac{(1-r)(2-r)}{1-r} = 2 - r$ OE		oe shown

9709/13/M/J/17

Question 63:

(a) A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The nth term of a progression is $p + qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n. [3]

(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of p and q.

Answer

Question	Answer	Marks	Guidance
8(a)	$ar = 12$ and $\frac{a}{1-r} = 54$	B1 B1	CAO, OE CAO, OE
	Eliminates a or r $\rightarrow$ $9r^2 - 9r + 2 = 0$ or $a^2 - 54a + 648 = 0$	M1	Elimination leading to a 3-term quadratic in a or r
	$\rightarrow r = \frac{2}{3}$ or $\frac{1}{3}$ hence to a $\rightarrow a = 18$ or 36	A1	Needs both values.
		4	
8(b)	nth term of a progression is $p + qn$		
8(b)(i)	first term $= p + q$. Difference $= q$ or last term $= p + qn$	B1	Need first term and, last term or common difference
	$S_n = \frac{n}{2}(2(p+q)+(n-1)q)$ or $\frac{n}{2}(2p+q+nq)$	M1A1	Use of S_n formula with their a and d. ok unsimplified for A1.
		3	
8(b)(ii)	Hence $2(2p+q+4q)=40$ and $3(2p+q+6q)=72$	DM1	Uses their S_n formula from (i)
	Solution $\rightarrow p = 5$ and $q = 2$ [Could use S_n with a and d $\rightarrow a = 7, d = 2 \rightarrow p = 5, q = 2$.]	A1	Note: answers 7, 2 instead of 5, 2 gets M1A0 – must attempt to solve for M1

9709/11/M/J/18

Question 64:

The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

Answer

Answer	Marks	Guidance
$\left[\frac{a(1-r^n)}{1-r}\right][\div]\left[\frac{a}{1-r}\right]$	M1M1	Correct formulae **used** with/without $r = 0.99$ or $n = 100$.
	DM1	Allow numerical a (M1M1). 3rd M1 is for division $\frac{S_n}{S_\infty}$ (or ratio) SOI
$1-0.99^{100}$ SOI OR $\frac{63(a)}{100(a)}$ SOI	A1	Could be shown multiplied by 100(%). Dep. on DM1
63(%) Allow 63.4 or 0.63 but not 2 infringements (e.g. 0.634, 0.63%)	A1	n = 99 used scores Max M3. Condone $a = 0.99$ throughout $S_n = S_\infty$ (without division shown) scores 2 / 5

9709/13/M/J/18

Question 65:

A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

(ii) Find the total amount of salt obtained in the first 12 weeks after the change.

Answer:

Question	Answer	Marks	Guidance
(i)	$r = 1.02$ or $\frac{102}{100}$ used in a GP in some way.	B1	Can be awarded here for use in S_n formula.
	Amount in 12th week = $8000\,(\textit{their } r)^{11}$ or $(\textit{their a from } \frac{8000}{\textit{their } r})(\textit{their } r)^{12}$	M1	Use of ar^{n-1} with a = 8000 & n = 12 or with a = $\frac{8000}{1.02}$ and n = 13.
	= 9950 (kg) awrt	A1	Note: Final answer of either 9943 or 9940 implies M1. Full marks can be awarded for a correct answer from a list of terms.
		3	
(ii)	In 12 weeks, total is $\frac{8000\left((\textit{their } r)^{12}-1\right)}{\left((\textit{their } r)-1\right)}$	M1	Use of S_n with a = 8000 and n = 12 or addition of 12 terms.
	= 107000 (kg) awrt	A1	Correct answer but no working 2/2

9709/12/M/J/18

Question 66:

(i) The first and second terms of a geometric progression are p and $2p$ respectively, where p is a positive constant. The sum of the first n terms is greater than $1000p$. Show that $2^n > 1001$. [2]

(ii) In another case, p and $2p$ are the first and second terms respectively of an arithmetic progression. The nth term is 336 and the sum of the first n terms is 7224. Write down two equations in n and p and hence find the values of n and p. [5]

Answer

Question	Answer	Marks	Guidance
(i)	$S_n = \frac{p(2^n-1)}{2-1}$ soi	M1	
	$p(2^n-1) > 1000p \rightarrow 2^n > 1001$ AG	A1	
		2	
(ii)	$p+(n-1)p = 336$	B1	Expect $np = 336$
	$\frac{n}{2}[2p+(n-1)p] = 7224$	B1	Expect $\frac{n}{2}(p+np) = 7224$
	Eliminate n or p to an equation in one variable	M1	Expect e.g. $168(1+n) = 7224$ or $1 + 336/p = 43$ etc
	$n = 42, p = 8$	A1A1	
		5	

9709/12/F/M/19

Question 67:

(a) The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period.

Scheme A ....

Scheme B ....

Answer:

	Answer	Marks	Guidance
(a)	$ar^2 = 48$, $ar^3 = 32$, $r = ⅔$ **or** $a = 108$	M1	Solution of the 2 eqns to give r (or a). A1 (both)
	$r = ⅔$ **and** $a = 108$	A1	
	$S\infty = \dfrac{108}{\frac{1}{3}} = 324$	A1	**FT** Needs correct formula and r between −1 and 1.
		3	
(b)	Scheme A $a = 2.50$, $d = 0.16$ $S_n = 12(5 + 23\times0.16)$	M1	Correct use of either AP S_n formula.
	$S_n = 104$ tonnes.	A1	
	Scheme B $a = 2.50$, $r = 1.06$	B1	Correct value of r used in GP.
	$= \dfrac{2.5(1.06^{24}-1)}{1.06-1}$	M1	Correct use of either S_n formula.
	$S_n = 127$ tonnes.	A1	
		5	

9709/11/M/J/19

Question 68:

(a) In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a.

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

(ii) Given that the tenth term is 36 more than the fourth term, find the value of a. [2]

(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

Answer:

Question	Answer	Marks	Guidance
10(a)(i)	$S_{10} = S_{15} - S_{10}$ or $S_{10} = S_{(11\text{ to }15)}$	M1	Either statement seen or implied.
	$5(2a + 9d)$ oe	B1	
	$7.5(2a + 14d) - 5(2a + 9d)$ or $\frac{5}{2}[(a + 10d) + (a+14d)]$ oe	A1	
	$d = \frac{a}{3}$ **AG**	A1	Correct answer from convincing working
		4	Condone starting with $d = \frac{a}{3}$ and evaluating both summations as 25a.
10(a)(ii)	$(a+9d) = 36 + (a+3d)$	M1	Correct use of $a+(n-1)d$ twice and addition of ±36
	$a = 18$	A1	

10(b)	$S_\infty = 9 \times S_4$; $\frac{a}{1-r} = 9\frac{a(1-r^4)}{1-r}$ or $9(a + ar + ar^2 + ar^3)$	B1	May have 12 in place of a.
	$9(1 - r^n) = 1$ where $n = 3, 4$ or 5	M1	Correctly deals with a and correctly eliminates '$1 - r$'
	$r^4 = \frac{8}{9}$ oe	A1	
	(5^{th} term =) $10\frac{2}{3}$ or 10.7	A1	
		4	Final answer of 10.6 suggests premature approximation – award 3/4 www.

9709/12/M/J/19

Question 69:

(b) The first, second and third terms of a geometric progression are x, $x - 3$ and $x - 5$ respectively.

(i) Find the value of x. [2]
(ii) Find the fourth term of the progression. [2]
(iii) Find the sum to infinity of the progression. [2]

Answer:

(b)(i)	$\frac{x-3}{x} = \frac{x-5}{x-3}$ oe (or use of a, ar and ar^2)	M1	Any valid method to obtain an equation in one variable.
	($a =$ or $x =$) 9	A1	
		2	
(b)(ii)	$r = \left(\frac{x-3}{x}\right)$ or $\left(\frac{x-5}{x-3}\right)$ or $\sqrt{\frac{x-5}{x}} = \frac{2}{3}$. Fourth term $= 9 \times (\frac{2}{3})^3$	M1	Any valid method to find r and the fourth term with *their* a & r.
	$2\frac{2}{3}$ or 2.67	A1	OE, AWRT
(b)(iii)	$S\infty = \frac{a}{1-r} = \frac{9}{1-\frac{2}{3}}$	M1	Correct formula and using *their* 'r' and 'a', with $\lvert r\rvert<1$, to obtain a numerical answer.
	27 or 27.0	A1	AWRT

9709/12/O/N/19

Question 70:

The first, second and third terms of a geometric progression are $3k$, $5k - 6$ and $6k - 4$, respectively.

(i) Show that k satisfies the equation $7k^2 - 48k + 36 = 0$. [2]
(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of k. [4]
(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

Answer:

9(i)	$\frac{5k-6}{3k} = \frac{6k-4}{5k-6} \rightarrow (5k-6)^2 = 3k(6k-4)$	M1	OR any valid relationship
	$25k^2 - 60k + 36 = 18k^2 - 12k \rightarrow 7k^2 - 48k + 36$	A1	AG
9(ii)	$k = \frac{6}{7}, 6$	B1B1	Allow 0.857(1) for $\frac{6}{7}$
	When $k = \frac{6}{7}$, $r = -\frac{2}{3}$	B1	Must be exact
	When $k = 6$, $r = \frac{4}{3}$	B1	
9(iii)	Use of $S_\infty = \frac{a}{1-r}$ with $r = their -\frac{2}{3}$ and $a = 3 \times their \frac{6}{7}$	M1	Provided $0 < \lvert their -2/3\rvert < 1$
	$\frac{18}{7} \div \left(1 + \frac{2}{3}\right) = \frac{54}{35}$ or 1.54	A1	FT if 0.857(1) has been used in part (ii).

Question 71:

(a) An arithmetic progression has a first term of 5 and a common difference of –3.

Find the number of terms such that the sum to n terms is first less than –200. [4]

(b) A geometric progression is such that its 3rd term is equal to $\frac{81}{64}$ and its 5th term is equal to $\frac{729}{1024}$.

(i) Find the first term of this progression and the positive common ratio of this progression. [5]

(ii) Hence find the sum to infinity of this progression. [1]

Answer:

Question	Answer	Marks	Partial Marks
10(a)	$-200 > \frac{n}{2}(-10 + (n-1)(-3))$ leading to $3n^2 - 13n - 400\ (> 0)$ $n = 13.9...$ so 14th term needed	4	**M1** for attempt to use sum to n terms, allow use of = or $\leqslant$ or < **A1** for correct quadratic expression **DM1** for attempt to solve **A1** for correct conclusion
10(b)(i)	$ar^2 = \frac{81}{64}$ $ar^4 = \frac{729}{1024}$ $r^2 = \frac{9}{16}$ $r = \frac{3}{4}$ $a = \frac{9}{4}$	5	**B1** for 3rd term **B1** for 5th term **M1** for attempt to solve their equations to obtain either r or a **A1** for r **A1** for a
10(b)(ii)	$S_\infty = 9$	B1	**FT** on *their* a and r, provided $\lvert r\rvert < 1$

4037/01/SP/20

Inequalities Past Papers 1972-2019

1. Find the solution set of the inequality $\frac{x-5}{1-x} > 1$. (J72/P2/1)

2. Find the solution set of the inequality $\frac{10}{x} < 19 - 6x$. (N72/P1/2)

3. Find the solution set of the inequality $\frac{4-x}{x-2} > 3$. (J73/P2/1)

4. In answering either part of this question, you may, if you wish, make use of rough sketch graphs.
   (a) Given that the inequalities $x + y > 1$, $3y > 2x - 1$, $3x > 2y$ are simultaneously satisfied, find the range of values to which x is restricted and the range of values to which y is restricted.
   (b) Find the solution set of the inequality $x + 1 < \frac{2}{x}$. (N73/P1/2)

5. Find the solution set of the inequality $\frac{12}{x-3} < x + 1$. (J74/P1/2)

6. (a) Calculate the area of the region of the $x - y$ plane defined by the simultaneous inequalities $y \leq 2x$, $x + y \leq 6$, $x \leq 5y$.
   (b) Find the solution set of the inequality $\frac{x-1}{x+1} > \frac{x}{6}$. (N74/P2/2)

7. Prove that the simultaneous inequalities $y < x$, $2y + x > 0$, $x + y < 12$ together imply $0 < x < 24$, and find the set of values to which y is restricted. (J75/P2/2)

8. Find the solution set of the inequalities
   (a) $\frac{3}{2} < \frac{2x-3}{x-5}$, (b) $3x(x-5) < 2(2x-3)$. (N75/P2/1)

9. Use a graphical method to find all pairs of positive integers (x, y) satisfying the following three inequalities simultaneously: $4x - y > 2$, $2x + y < 12$, $y > x$. (J76/P2/2)

10. Draw a diagram illustrating the region S of the $x - y$ plane which is defined by the simultaneous inequalities $x + y \geq 7$, $2x + y \leq 13$, $2x + 3y \leq 19$, and give the coordinates of the vertices of S. Prove that, if the line $y = kx$ intersects S, then $\frac{1}{6} \leq k \leq 2\frac{1}{2}$. The point P lies on $y = kx$ and is in the region S. Prove that, when $\frac{1}{6} \leq k \leq \frac{3}{5}$, the maximum value for the y-coordinate of P is $13k/(2 + k)$, and find the corresponding expression when $\frac{3}{5} \leq k \leq 2\frac{1}{2}$. (J77/P1/2)

11. Find the solution set of the inequality $\frac{1}{x-1} < \frac{1}{x+1}$ where $x \in R$, $x \neq 1, -1$. (J78/P2/3)

12. The set S is $\{(x, y): y + 2x \geq 5$ and $x^2 + y^2 \leq 10$, $(x, y) \in R \times R\}$. Give a sketch of the $x - y$ plane to show the region in which the points representing the members of S must lie. If $(x, kx) \in S$ find the set of possible values of k. (N78/P1/1)

13. (a) Show that the region of the $x - y$ plane within which the following four simultaneous inequalities are satisfied is, in fact, defined by only three of the inequalities, and state the redundant inequality. $x < 6$, $y < x$, $3x - y > 3$, $x + 2y > 6$.
    (b) Find the solution set of the inequality $\frac{2}{x+1} > x$, where $x \in R$, $x \neq -1$. (J79/P2/2)

14. Solve the inequality $\frac{15}{x-2} > 3x - 2$, $(x \in R, x \neq 2)$. (N79/P1/1)

15. (a) Find the solution set for x given that the following three relations for x, y, where $x, y \in R$, are simultaneously true: $y < x + 1$, $y + 6x < 20$, $x = 5y - 7$.
    (b) Find the solution set of the inequality $\frac{12}{x-3} < x + 1$, $(x \in R, x \neq 3)$. (J80/P2/2)

16. (a) Functions f and g are defined by $f : x \to x(x-1)$, $g : x \to (x-1)(3x-5)$, where $x \in R$ in each case.

(i) Find the solution set, S, of the inequality $f(x) \geq g(x)$.

(ii) Sketch the graph of $y = f(x) - g(x)$ for $x \in S$, and state the greatest and least values of $f(x) - g(x)$ for $x \in S$.

(b) Illustrate by means of a sketch the subset of the x-y plane given by $\{(x, y) : |x| < \lfloor y \rfloor\}$. (J81/P1/2)

17. (a) The set S of ordered pairs of real numbers is given by $S = \{(x, y): y \geq 3x, y + 2x \leq 35, y \leq x^2\}$. Draw a sketch showing, by shading, the region of the x-y plane containing all the points (x, y) in S. Given that $(x, y) \in S$, find the maximum value of $(4x - y)$ and the minimum value of $(x + y)$.

(b) Find the solution set of the inequality $\frac{6}{x-1} > x$, $(x \in R, x \neq 1)$. (N81/P1/2)

18. Solve the inequality $\frac{2x}{x+1} > x$, $(x \in R, x \neq -1)$. (J82/P2/1)

19. Find the set of values of k for which, for all real values of x, $3x^2 + 3x + k > 0$ and $3x^2 + kx + 3 > 0$. (N82/P1/2)

20. Find the solution set of the inequality $\frac{1}{2-x} < \frac{1}{x-3}$. (N82/P2/1)

21. (a) Find the solution set of the inequality $\frac{x-1}{x+1} + 1 > 0$.

(b) Given that $x + 2y \geq 3$ and $y - 3x \geq 5$, show that $y - x \geq 3$. (J83/P2/3)

22. The set, S, of ordered pairs, (x, y), of real numbers is defined by $S = \{(x, y) : y - 2x \leq 0, 2y - x \geq 0, 2y + x - 20 \leq 0\}$. Illustrate the region in the x-y plane determined by the set S. For $(x, y) \in S$ find

(a) the greatest value of $x + 4y$,

(b) the greatest value of $x^2 + y^2$,

(c) the set of values of $y^2 - 6y$. (J83/P1/2)

23. (a) Solve the inequality $\frac{3}{1-x} < 5 - 4x$. $(x \in R, x \neq 1)$.

(b) Draw a sketch to illustrate the region R of the x-y plane defined by the simultaneous inequalities $3x - 7y \geq 1$, $2x + y \leq 12$. Show that the line $y = mx + (2 - 5m)$ passes through the vertex of R for all values of m. Deduce the set of values of m for which the inequality $y \leq mx + (2 - 5m)$ is true for all (x, y) in R. (J84/P2/3)

24. (a) Illustrate the solution set of the simultaneous inequalities $9 \leq y + 3x \leq 18$, $0 \leq 2y - 3x \leq 18$ by means of a diagram, and write down the sets of values to which x and y are separately restricted.

(b) Find the solution set of the inequality $\frac{2}{x-3} > \frac{3}{x-2}$, where $x \in R$, $x \neq 2$, $x \neq 3$. (J85/P2/2)

25. Find the solution set of the inequalities:

(a) $|x - 2| < 2x$ (b) $\frac{6}{|x|+1} < |x|$ (c) $x^2 - 6x + 7 > 0$

26. Find the range(s) of the values of x for which the following inequalities hold:

(a) $|3x + 5| < 4$ (c) $|x^2 + 1| < |x^2 - 9|$

(b) $\left|\frac{2x+1}{x-3}\right| > 1$ (d) $|3 - 2x| \leq |x + 4|$

27. (a) Find the solution set of the inequality $x + \frac{1}{x} > \frac{5}{2}$.

(b) Solve the simultaneous equations $x + y = 6$, $x^2 + y^2 = 26$. Hence, with the aid of a diagram, or otherwise, determine all the pairs of integers (x, y) for which the inequations $x + y \geq 6$, $x^2 + y^2 \leq 26$ are simultaneously true. (N85/P1/2)

28. Solve the following inequalities:

(a) $\frac{x+1}{x-1} < 4$, (b) $\frac{|x|+1}{|x|-1} < 4$, (c) $\left|\frac{x+1}{x-1}\right| < 4$. (J87/P1/18)

29. Sketch the graph of $y = |x + 2|$ and hence, or otherwise, solve the inequality $|x + 2| > 2x + 1$, $x \in R$. (N87/P1/4)

30. Solve the inequality $x - x^3 > 0$. (J88/P2/5)

31. (a) Sketch, on the same diagram, the graphs of $y = \frac{1}{x}$ and $y = x - \frac{3}{2}$. Find the solution set of the inequality $x - \frac{3}{2} > \frac{1}{x}$.

(b) Sketch, on separate diagrams, the graphs of $y = |x|$, $y = |x - 3|$, $y = |x - 3| + |x + 3|$. Find the solution set of the equation $|x - 3| + |x + 3| = 6$. (J89/P1/14)

32. Solve the inequality $x(x - 1)(x - 2) > 0$. (N89/P1/6)

33. Solve, for $x \in R$, each of the following inequalities:

(a) $\frac{x}{x-2} < 5$, (b) $x(x - 2) < 5$, (c) $|x| < 4\,|x - 3|$. (J90/P1/15)

34. Solve the equation $4|x| = |x - 1|$. On the same diagram, sketch the graphs of $y = 4\,|x|$ and $y = |x - 1|$ and hence, or otherwise, solve the inequality $4\,|x| > |x - 1|$. (N90/P1/4)

35. Solve the inequality $x^3 < 6x - x^2$. (J91/P1/3)

36. Solve the inequality $\frac{x+5}{2-x} < 3$. (J92/P1/4)

37. Sketch, on a single diagram, the graphs of $x + 2y = 6$ and $y = |x + 2|$. Hence, or otherwise, solve the inequality $|x + 2| < \frac{1}{2}(6 - x)$. (N92/P1/4)

38. Express $3x^2 - 12x - 4$ in the form $A(x + B)^2 + C$, giving the values of A, B and C. Hence or otherwise solve the inequality $3x^2 - 12x - 4 > 0$. Give your answer in a form involving one or more of the intervals $x < a$, $x > b$, $a < x < b$, where the value of a and b contain surds. Hence or otherwise solve exactly the inequalities

(a) $3y^2 + 12y - 4 > 0$,

(b) $3e^{2u} - 12e^{u} - 4 > 0$. (N93/P1/12)

Question 39:

Solve the inequality $|x + 2| < |5 - 2x|$. [4]

Answer:

EITHER:	State or imply non-modular inequality $(x+2)^2 < (5-2x)^2$, or corresponding equation	B1
	Expand and make reasonable solution attempt at 2- or 3-term quadratic, or equivalent	M1
	Obtain critical values 1 and 7	A1
	State correct answer $x < 1$, $x > 7$	A1
OR:	State one correct equation for a critical value e.g. $x + 2 = 5 - 2x$	M1
	State two relevant equations separately e.g. $x + 2 = 5 - 2x$ and $x + 2 = -(5 - 2x)$	A1
	Obtain critical values 1 and 7	A1
	State correct answer $x < 1$, $x > 7$	A1

OR: State one critical value (probably $x = 1$), from a graphical method or by inspection or by solving a linear inequality B1
State the other critical value correctly B2
State correct answer $x < 1$, $x > 7$ B1 **4**
[The answer $7 < x < 1$ scores B0.]

9709/2/M/J/02

Question 40:

Solve the inequality $|9 - 2x| < 1$. [3]

Answers:

EITHER: State or imply non-modular inequality $(9-2x)^2 < 1$, or a correct pair of linear inequalities, combined or separate, e.g. $-1 < 9 - 2x < 1$ B1
Obtain both critical values 4 and 5 B1
State correct answer $4 < x < 5$; accept $x > 4$, $x < 5$ B1

OR: State a correct equation or pair of equations for both critical values e.g. $9 - 2x = 1$ and $9 - 2x = -1$, or $9 - 2x = \pm 1$ B1
Obtain critical values 4 and 5 B1
State correct answer $4 < x < 5$; accept $x > 4$, $x < 5$ B1

OR: State one critical value (probably $x = 4$) from a graphical method or by inspection or by solving a linear inequality or equation B1
State the other critical value correctly B1
State correct answer $4 < x < 5$; accept $x > 4$, $x < 5$ B1 **3**
[Use of $\le$, throughout, or at the end, scores a maximum of B2.]

9709/3/O/N/02

Question 41:

Solve the inequality $|x - 2| < 3 - 2x$. [4]

Answers:

EITHER State or imply non-modular inequality $(x-2)^2 < (3-2x)^2$, or corresponding equation B1
Expand and make a reasonable solution attempt at a 2- or 3-term quadratic, or equivalent M1
Obtain critical value $x = 1$ A1
State answer $x < 1$ only A1

OR State the relevant linear equation for a critical value, i.e. $2 - x = 3 - 2x$, or equivalent B1
Obtain critical value $x = 1$ B1
State answer $x < 1$ B1
State or imply by omission that no other answer exists B1

OR Obtain the critical value $x = 1$ from a graphical method, or by inspection, or by solving a linear inequality B2
State answer $x < 1$ B1
State or imply by omission that no other answer exists B1

[4] **9709/3/M/J/03**

Question 42:

Solve the inequality $|x - 4| > |x + 1|$. [4]

Answers:

EITHER: State or imply non-modular inequality $(x-4)^2 > (x+1)^2$, or corresponding equation B1
Expand and solve a linear inequality, or equivalent M1
Obtain critical value $1\frac{1}{2}$ A1
State correct answer $x < 1\frac{1}{2}$ (allow $\le$) A1

OR: State a correct linear equation for the critical value e.g. $4 - x = x + 1$ B1
Solve the linear equation for x M1
Obtain critical value $1\frac{1}{2}$, or equivalent A1
State correct answer $x < 1\frac{1}{2}$ A1

OR: State the critical value $1\frac{1}{2}$, or equivalent, from a graphical method or by inspection or by solving a linear inequality B3
State correct answer $x < 1\frac{1}{2}$ B1

9709/2/M/J/03

Question 43:

Find the set of values of x satisfying the inequality $|8 - 3x| < 2$. [3]

Answers:

1 *EITHER:* State or imply non-modular inequality e.g. $-2 < 8-3x < 2$, or $(8-3x)^2 < 2^2$, or corresponding equation or pair of equations M1
Obtain critical values 2 and $3\frac{1}{3}$ A1
State correct answer $2 < x < 3\frac{1}{3}$ A1

OR: State one critical value (probably $x = 2$), from a graphical method or by inspection or by solving a linear equality or equation B1
State the other critical value correctly B1
State correct answer $2 < x < 3\frac{1}{3}$ B1

[3] **9709/02/O/N/03**

Question 44:

Solve the inequality $|2^x - 8| < 5$. [4]

Answers:

EITHER: State or imply non-modular inequality $-5 < 2^x - 8 < 5$, or $(2^x - 8)^2 < 5^2$ or corresponding pair of linear equations or quadratic equation B1
Use correct method for solving an equation of the form $2^x = a$ M1
Obtain critical values 1.58 and 3.70, or exact equivalents A1
State correct answer $1.58 < x < 3.70$ A1

OR: Use correct method for solving an equation of the form $2^x = a$ M1
Obtain one critical value (probably 3.70), or exact equivalent A1
Obtain the other critical value, or exact equivalent A1
State correct answer $1.58 < x < 3.70$ A1

[Allow 1.59 and 3.7. Condone $\leq$ for $<$. Allow final answers given separately. Exact equivalents must be in terms of ln or logarithms to base 10.]
[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

9709/03/O/N/03

Question 45:

Solve the inequality $|2x + 1| < |x|$. [4]

Answers:

EITHER: State or imply non-modular inequality $(2x+1)^2 < x^2$ or corresponding quadratic equation or pair of linear equations $(2x + 1) = \pm x$ B1
Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
Obtain critical values $x = -1$ and $x = -\frac{1}{3}$ only A1
State answer $-1 < x < -\frac{1}{3}$ A1

OR: Obtain the critical value $x = -1$ from a graphical method , or by inspection, or by solving a linear inequality or equation — B1

Obtain the critical value $x = -\frac{1}{3}$ (deduct B1 from B3 if extra values are obtained) — B2

State answer $-1 < x < -\frac{1}{3}$ — B1 **4**

[Condone $\le$ for $<$; accept -0.33 for $-\frac{1}{3}$.]

9709/03/M/J/04

Question 46:

Solve the inequality $|x+1| > |x|$. [3]

Answers:

EITHER: State or imply non-modular inequality $(x + 1)^2 > x^2$ or corresponding quadratic equation or linear equation $x + 1 = -x$ — B1

Obtain critical value $-\frac{1}{2}$ — B1

State answer $x > -\frac{1}{2}$ — B1

OR: Obtain critical value $-\frac{1}{2}$ by solving a linear inequality or by graphical method or inspection — B2

State answer $x > -\frac{1}{2}$ — B1 **3**

[For $2x + 1 > 0$, $x > +\frac{1}{2}$, or similar reasonable method] — M1

9709/02/O/N/04

Question 47:

Solve the inequality $|x| > |3x-2|$. [4]

Answers:

EITHER State or imply non-modular inequality $x^2 > (3x-2)^2$, or corresponding equation
Expand and make reasonable solution attempt at 2- or 3-term quadratic, or equivalent
Obtain critical values ½ and 1
State correct answer $\frac{1}{2} < x < 1$

OR State one correct linear equation for a critical value
State two equations separately
Obtain critical values ½ and 1
State correct answer $\frac{1}{2} < x < 1$

OR State one critical value from a graphical method or inspection or by solving a linear inequality
State the other critical value correctly
State correct answer $\frac{1}{2} < x < 1$

9709/02/M/J/05

Question 48:

Given that a is a positive constant, solve the inequality

$$|x-3a| > |x-a|.$$ [4]

Answer:

EITHER: State or imply non-modular inequality $(x-3a)^2 > (x-a)^2$, or corresponding equation — B1
Expand and solve the inequality, or equivalent — M1
Obtain critical value $2a$ — A1
State correct answer $x < 2a$ only — A1

OR: State a correct linear equation for the critical value, e.g. $x - 3a = -(x - a)$, or corresponding inequality — B1
Solve the linear equation for x, or equivalent — M1
Obtain critical value $2a$ — A1
State correct answer $x < 2a$ only — A1

OR: Make recognizable sketches of both $y = |x - 3a|$ and $y = |x - a|$ on a single diagram — B1
Obtain a critical value from the intersection of the graphs — M1
Obtain critical value $2a$ — A1
Obtain correct answer $x < 2a$ only — A1

9709/03/O/N/05

Question 49:

Solve the inequality $2x > |x - 1|$. [4]

Answer:

EITHER:	State or imply non-modular inequality $(2x)^2 > (x-1)^2$, or corresponding equation	B1
	Expand and make a reasonable solution attempt at a 2- or 3-term quadratic	M1
	Obtain critical value $x = \frac{1}{3}$	A1
	State answer $x > \frac{1}{3}$ only	A1
OR:	State the relevant critical linear equation, i.e. $2x = 1 - x$	B1
	Obtain critical value $x = \frac{1}{3}$	B1
	State answer $x > \frac{1}{3}$	B1
	State or imply by omission that no other answer exists	B1
OR:	Obtain the critical value $x = \frac{1}{3}$ from a graphical method, or by inspection, or by solving a linear inequality	B2
	State answer $x > \frac{1}{3}$	B1
	State or imply by omission that no other answer exists	B1

9709/03/M/J/06

Question 50:

Solve the inequality $|2x - 1| > |x|$. [4]

Answers:

EITHER:	State or imply non-modular inequality $(2x-1)^2 > x^2$ or corresponding quadratic equation or pair of linear equations $2x - 1 = \pm x$	M1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values $x = 1$ and $x = \frac{1}{3}$	A1
	State answer $x < \frac{1}{3}, x > 1$	A1
OR:	Obtain critical value $x = 1$ from a graphical method, or by inspection, or by solving a linear inequality or linear equation	B1
	Obtain the critical value $x = \frac{1}{3}$ similarly	B2
	State answer $x < \frac{1}{3}, x > 1$	B1

9709/02/O/N/06

Question 51:

(i) Express 4^x in terms of y, where $y = 2^x$. [1]

(ii) Hence find the values of x that satisfy the equation

$$3(4^x) - 10(2^x) + 3 = 0,$$

giving your answers correct to 2 decimal places. [5]

Answers:

(i)	State or imply that $4^x = y^2 \quad (=2^{2x})$		B1
(ii)	Carry out recognizable solution method for a quadratic equation in y		M1
	Obtain $y = 3$ and $y = \frac{1}{3}$ from $3y^2 - 10y + 3 = 0$		A1
	Use logarithmic method to solve an equation of the form $2^x = k$, where $k > 0$		M1
	State answer 1.58		A1
	State answer -1.58	(A1 $\sqrt{}$ if ± 1.59)	A1

9709/02/O/N/06

Question 52:

Solve the inequality $|x-3| > |x+2|$. [4]

Answers:

EITHER	State or imply non-modular inequality $(x-3)^2 > (x+2)^2$, or corresponding equation	M1
	Expand and solve a linear inequality, or equivalent	M1
	Obtain critical value $\frac{1}{2}$	A1
	State correct answer $x < \frac{1}{2}$ (allow $x \leqslant \frac{1}{2}$)	A1
OR	State a correct linear equation for the critical value, e.g. $3-x = x+2$, or corresponding correct inequality, e.g. $-(x-3) > (x+2)$	M1
	Solve the linear equation, or inequality	M1
	Obtain critical value $\frac{1}{2}$	A1
	State correct answer $x < \frac{1}{2}$	A1
OR	Make recognisable sketches of both $y = \lvert x-3 \rvert$ and $y = \lvert x+2 \rvert$ on a single diagram	B1
	Obtain a critical value from the intersection of the graphs	M1
	Obtain critical value $\frac{1}{2}$	A1
	State final answer $x < \frac{1}{2}$	A1

9709/02/M/J/07

Question 53:

(i) Solve the inequality $|y-5| < 1$. [2]

(ii) Hence solve the inequality $|3^x - 5| < 1$, giving 3 significant figures in your answer. [3]

Answers:

(i)	Obtain critical values 4 and 6	B1	
	State answer $4 < y < 6$	B1	[2]
(ii)	Use correct method for solving an equation of the form $3^x = a$, where $a > 0$	M1	
	Obtain one critical value, i.e. either 1.26 or 1.63	A1	
	State answer $1.26 < x < 1.63$	A1	[3]

9709/02/O/N/07

Question 54:

Solve the inequality $|x-2| > 3|2x+1|$. [4]

Answers:

EITHER	State or imply non-modular inequality $(x-2)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(x-2) = \pm 3(2x+1)$	B1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values $x = -1$ and $x = -\frac{1}{7}$	A1
	State answer $-1 < x < -\frac{1}{7}$	A1

OR	Obtain the critical value $x = -1$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1
	Obtain the critical value $x = -\frac{1}{7}$ similarly	B2
	State answer $-1 < x < -\frac{1}{7}$	B1
	[Do not condone $\leq$ for $<$; accept $-\frac{5}{35}$ and -0.14 for $-\frac{1}{7}$.]	

9709/03/M/J/08

Question 55:

Solve the inequality $|x-3| > |2x|$. [4]

Answers:

EITHER:	State or imply non-modular inequality $(x-3)^2 > (2x)^2$ or corresponding quadratic equation or pair of linear equations $(x-3) = \pm 2x$	M1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values $x = 1$ and $x = -3$	A1
	State answer $-3 < x < 1$	A1
OR:	Obtain critical value $x = -3$ from a graphical method, or by inspection, or by solving a linear inequality or linear equation	B1
	Obtain the critical value $x = 1$ similarly	B2
	State answer $-3 < x < 1$	B1

9709/02/O/N/08

Question 56:

Solve the inequality $|3x+2| < |x|$. [4]

Answers:

EITHER:	State or imply non-modular inequality $(3x+2)^2 < x^2$, or corresponding quadratic equation, or pair of linear equations $3x+2 = \pm x$	M1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values $x = -1$ and $x = -\frac{1}{2}$	A1
	State answer $-1 < x < -\frac{1}{2}$	A1
OR:	Obtain the critical value $x = -1$ from a graphical method or by inspection, or by solving a linear equation or inequality	B1
	Obtain the critical value $x = -\frac{1}{2}$ similarly	B2
	State answer $-1 < x < -\frac{1}{2}$	B1

9709/02/M/J/09

Question 57:

Solve the inequality $2 - 3x < |x-3|$. [4]

Answers:

EITHER:	State or imply non-modular inequality $(2-3x)^2 < (x-3)^2$, or corresponding equation, and make a reasonable solution attempt at a 3-term quadratic	M1
	Obtain critical value $x = -\frac{1}{2}$	A1
	Obtain $x > -\frac{1}{2}$	A1
	Fully justify $x > -\frac{1}{2}$ as only answer	A1
*OR*1:	State the relevant critical linear equation, i.e. $2 - 3x = 3 - x$	B1
	Obtain critical value $x = -\frac{1}{2}$	B1
	Obtain $x > -\frac{1}{2}$	B1
	Fully justify $x > -\frac{1}{2}$ as only answer	B1
*OR*2:	Obtain the critical value $x = -\frac{1}{2}$ by inspection, or by solving a linear inequality	B2
	Obtain $x > -\frac{1}{2}$	B1
	Fully justify $x > -\frac{1}{2}$ as only answer	B1

*OR*3:	Make recognisable sketches of $y = 2 - 3x$ and $y = \lvert x - 3\rvert$ on a single diagram	B1
	Obtain critical value $x = -\frac{1}{2}$	B1
	Obtain $x > -\frac{1}{2}$	B1
	Fully justify $x > -\frac{1}{2}$ as only answer	B1
	[Condone $\geqslant$ for $>$ in the third mark but not the fourth.]	

9709/31/O/N/09

Question 58:

Solve the equation $3^{x+2} = 3^x + 3^2$, giving your answer correct to 3 significant figures. [4]

Answers:

EITHER:	Use laws of indices correctly and solve a linear equation for 3^x, or for 3^{-x}	M1
	Obtain 3^x, or 3^{-x} in any correct form, e.g. $3^x = \dfrac{3^2}{(3^2-1)}$	A1
	Use correct method for solving $3^{\pm x} = a$ for x, where $a > 0$	M1
	Obtain answer $x = 0.107$	A1
OR:	State an appropriate iterative formula, e.g. $x_{n+1} = \dfrac{\ln(3^{x_n} + 9)}{\ln 3} - 2$	B1
	Use the formula correctly at least once	M1
	Obtain answer $x = 0.107$	A1
	Show that the equation has no other root but 0.107	A1
	[For the solution 0.107 with no relevant working, award B1 and a further B1 if 0.107 is shown to be the only root.]	

9709/31/O/N/09

Question 59:

Solve the inequality $\lvert x + 3a\rvert > 2\lvert x - 2a\rvert$, where a is a positive constant. [4]

Answers:

EITHER:	State or imply non-modular inequality $(x+3a)^2 > (2(x-2a))^2$, or corresponding quadratic equation, or pair of linear equations $(x+3a) = \pm 2(x-2a)$	B1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values $x = \frac{1}{3}a$ and $x = 7a$	A1
	State answer $\frac{1}{3}a < x < 7a$	A1
OR:	Obtain the critical value $x = 7a$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1
	Obtain the critical value $x = \frac{1}{3}a$ similarly	B2
	State answer $\frac{1}{3}a < x < 7a$	B1
	[Do not condone $\leqslant$ for $<$; accept 0.33 for $\frac{1}{3}$.]	

9709/31/M/J/10

Question 60:

Solve the equation

$$\frac{2^x + 1}{2^x - 1} = 5,$$

giving your answer correct to 3 significant figures. [4]

Answers:

EITHER:	Attempt to solve for 2^x	M1
	Obtain $2^x = 6/4$, or equivalent	A1
	Use correct method for solving an equation of the form $2^x = a$, where $a > 0$	M1
	Obtain answer $x = 0.585$	A1

OR:	State an appropriate iterative formula, e.g. $x_{n+1} = \ln((2^{x_n} + 6) / 5) / \ln 2$	B1	
	Use the iterative formula correctly at least once	M1	
	Obtain answer $x = 0.585$	A1	
	Show that the equation has no other root but 0.585	A1	[4]

[For the solution 0.585 with no relevant working, award B1 and a further B1 if 0.585 is shown to be the only root.]

9709/32/M/J/10

Question 61:

Solve the inequality $|x - 3| > 2|x + 1|$. [4]

Answers:

EITHER:	State or imply non-modular inequality $(x-3)^2 > (2(x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(x - 3) = \pm 2(x + 1)$	B1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values -5 and $\frac{1}{3}$	A1
	State answer $-5 < x < \frac{1}{3}$	A1
OR:	Obtain the critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1
	Obtain the critical value $x = \frac{1}{3}$ similarly	B2
	State answer $-5 < x < \frac{1}{3}$	B1
	[Do not condone $\leq$ for $<$; accept 0.33 for $\frac{1}{3}$.]	

9709/33/M/J/10

Question 62:

Solve the inequality $2|x - 3| > |3x + 1|$. [4]

Answers:

EITHER:	State or imply non-modular inequality $(2(x-3))^2 > (3x + 1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x - 3) = \pm(3x + 1)$	B1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values $x = -7$ and $x = 1$	A1
	State answer $-7 < x < 1$	A1
OR:	Obtain critical value $x = -7$ or $x = 1$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1
	Obtain critical values $x = -7$ and $x = 1$	B2
	State answer $-7 < x < 1$	B1
	[Do not condone: $\leq$ for $<$.]	

9709/32/O/N/10

Question 63:

Solve the inequality $|x| < |5 + 2x|$. [3]

Answers:

EITHER:	State or imply non-modular inequality $x^2 < (5+2x)^2$, or corresponding equation, or pair of linear equations $x = \pm(5 + 2x)$	M1
	Obtain critical values -5 and $-\frac{5}{3}$ only	A1
	Obtain final answer $x < -5$, $x > -\frac{5}{3}$	A1

OR:	State one critical value e.g. –5, by solving a linear equation or inequality, or from a graphical method, or by inspection	B1
	State the other critical value, e.g. $-\frac{5}{3}$, and no other	B1
	Obtain final answer $x < -5, x > -\frac{5}{3}$	B1
	[Do not condone ≤ or ≥.]	

9709/32/M/J/11

Question 64:

Solve the equation $|4 - 2^x| = 10$, giving your answer correct to 3 significant figures. [3]

Answer:

State or imply $4 - 2^x = -10$ and 10	B1
Use correct method for solving equation of form $2^x = a$	M1
Obtain 3.81	A1

9709/31/M/J/12

Question 65:

Find the set of values of x satisfying the inequality $3|x - 1| < |2x + 1|$. [4]

Answers:

EITHER	State or imply non-modular inequality $(3(x-1))^2 < (2x+1)^2$ or corresponding quadratic equation, or pair of linear equations $3(x-1) = \pm(2x+1)$	B1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain critical values $x = \frac{2}{5}$ and $x = 4$	A1
	State answer $\frac{2}{5} < x < 4$	A1
OR	Obtain critical value $x = \frac{2}{5}$ or $x = 4$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1
	Obtain critical values $x = \frac{2}{5}$ and $x = 4$	B2
	State answer $\frac{2}{5} < x < 4$	B1
	[Do not condone ≤ for <.]	

9709/32/O/N/12

Question 66:

Solve the equation

$$5^{x-1} = 5^x - 5,$$

giving your answer correct to 3 significant figures. [4]

Answer:

EITHER	Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1}	M1
	Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x = \frac{5}{1 - 1/5}$	A1
	Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a > 0$	M1
	Obtain answer $x = 1.14$	A1
OR	Use an appropriate iterative formula, e.g. $x_{n+1} = \frac{\ln(5^{x_n - 1} + 5)}{\ln 5}$, correctly, at least once	M1
	Obtain answer 1.14	A1
	Show sufficient iterations to at least 3 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1
	Show there is no other root	A1
	[For the solution $x = 1.14$ with no relevant working give B1, and a further B1 if 1.14 is shown to be the only solution.]	

9709/32/O/N/12

Question 67:

(i) Solve the equation $|4x - 1| = |x - 3|$. [3]

(ii) Hence solve the equation $|4^{y+1} - 1| = |4^y - 3|$ correct to 3 significant figures. [3]

Answers:

(i) Either State or imply non-modular equation $(4x-1)^2 = (x-3)^2$ or pair of linear equations $4x-1 = \pm(x-3)$ B1

Solve a three-term quadratic equation or two linear equations M1

Obtain $-\frac{2}{3}$ and $\frac{4}{5}$ A1

Or Obtain value $-\frac{2}{3}$ from inspection or solving linear equation B1

Obtain value $\frac{4}{5}$ similarly B2

(ii) State or imply at least $4^y = \frac{4}{5}$, following a positive answer from part (i) B1√

Apply logarithms and use $\log a^b = b\log a$ property M1

Obtain -0.161 and no other answer A1 9709/31/M/J/13

Question 68:

Solve the equation $2|3^x - 1| = 3^x$, giving your answers correct to 3 significant figures. [4]

Answers:

EITHER: State or imply non-modular equation $2^2(3^x-1)^2 = (3^x)^2$, or pair of equations $2(3^x-1) = \pm 3^x$ M1

Obtain $3^x = 2$ and $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$) A1

OR: Obtain $3^x = 2$ by solving an equation or by inspection B1

Obtain $3^x = \frac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection B1

Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$ M1

Obtain final answers 0.631 and -0.369 A1 9709/32/O/N/13

Question 69:

Solve the inequality $x^2 - x - 2 > 0$. [3]

Answer:

$(x+1)(x-2)$ or other valid method	M1	Attempt soln of eqn or other method
$-1, 2$	A1	
$x < -1, x > 2$	A1	Penalise $\le, \ge$
	[3]	

9709/13/O/N/13

Question 70:

Find the set of values of x satisfying the inequality

$$|x + 2a| > 3|x - a|,$$

where a is a positive constant. [4]

Answers:

EITHER: State or imply non-modular inequality $(x+2a)^2 > (3(x-a))^2$, or corresponding quadratic equation, or pair of linear equations $(x+2a) = \pm 3(x-a)$ B1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x M1

Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$ A1

State answer $\frac{1}{4}a < x < \frac{5}{2}a$ A1

OR: Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1

Obtain critical value $x = \frac{1}{4}a$ similarly B2

State answer $\frac{1}{4}a < x < \frac{5}{2}a$ B1

[Do not condone $\leqslant$ for $<$.]

9709/32/M/J/14

Question 71:

(i) Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

Answers:

	(i) $(2x-3)^2 - 9$	**B1B1** **[2]**	For -3 and -9
	(ii) $2x-3>4 \quad 2x-3<-4$	**M1**	At least one of these statements
	$x > 3\frac{1}{2}$ (*or*) $x < -\frac{1}{2}$ cao	**A1**	Allow 'and' $3\frac{1}{2}, -\frac{1}{2}$ soi scores first M1
	Allow $-\frac{1}{2} > x > 3\frac{1}{2}$		
OR	$4x^2 - 12x - 7 \to (2x-7)(2x+1)$	**M1**	Attempt to solve 3-term quadratic
	$x > 3\frac{1}{2}$ (*or*) $< -\frac{1}{2}$ cao	**A1** **[2]**	Allow 'and' $3\frac{1}{2}, -\frac{1}{2}$ soi scores first M1
	Allow $-\frac{1}{2} > x > 3\frac{1}{2}$		

9709/11/M/J/14

Question 72:

Solve the inequality $|3x - 1| < |2x + 5|$. [4]

Answers:

Either	State or imply non-modular inequality $(3x-1)^2 < (2x+5)^2$ or corresponding quadratic equation or pair of linear equations $3x - 1 = \pm(2x+5)$	B1
	Solve a three-term quadratic or two linear equations $5x^2 - 26x - 24 < 0$	M1
	Obtain $-\frac{4}{5}$ and 6	A1
	State $-\frac{4}{5} < x < 6$	A1
Or	Obtain value 6 from graph, inspection or solving linear equation	B1
	Obtain value $-\frac{4}{5}$ similarly	B2
	State $-\frac{4}{5} < x < 6$	B1

9709/33/O/N/14

Question 73:

Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

Answer:

Use laws of indices correctly and solve for u M1

Obtain u in any correct form, e.g. $u = \dfrac{16}{16-1}$ A1

Use correct method for solving an equation of the form $4^x = a$, where $a > 0$ M1

Obtain answer $x = 0.0466$ A1

9709/32/M/J/15

Question 74:

Solve the inequality $|x - 2| > 2x - 3$. [4]

Answers:

EITHER:	State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation	B1
	Solve a 3-term quadratic, as in Q1.	M1
	Obtain critical value $x = \frac{5}{3}$	A1
	State final answer $x < \frac{5}{3}$ only	A1
*OR*1:	State the relevant critical linear inequality $(2-x) > (2x-3)$, or corresponding equation	B1
	Solve inequality or equation for x	M1
	Obtain critical value $x = \frac{5}{3}$	A1
	State final answer $x < \frac{5}{3}$ only	A1
*OR*2:	Make recognisable sketches of $y = 2x-3$ and $y = \lvert x-2 \rvert$ on a single diagram	B1
	Find x-coordinate of the intersection	M1
	Obtain $x = \frac{5}{3}$	A1
	State final answer $x < \frac{5}{3}$ only	A1

9709/33/M/J/15

Question 75:

Solve the inequality $|2x - 5| > 3|2x + 1|$. [4]

Answers:

EITHER: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$	**B1**
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values -2 and $\frac{1}{4}$	**A1**
State final answer $-2 < x < \frac{1}{4}$	**A1**
OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**B1**
Obtain critical value $x = \frac{1}{4}$ similarly	**B2**
State final answer $-2 < x < \frac{1}{4}$	**B1**
[Do not condone $\leqslant$ for $<$]	

9709/32/O/N/15

Question 76:

Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

Answer:

State or imply $1 + u = u^2$	**B1**
Solve for u	**M1**
Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	**A1**
Use correct method for finding x from a positive root	**M1**
Obtain $x = 0.438$ and no other answer	**A1**

9709/32/O/N/15

Question 77:

(i) Solve the equation $2|x - 1| = 3|x|$. [3]

(ii) Hence solve the equation $2|5^x - 1| = 3|5^x|$, giving your answer correct to 3 significant figures. [2]

Answers:

(i) *EITHER*: State or imply non-modular equation $(2(x-1))^2=(3x)^2$, or pair of linear equations $2(x-1)=\pm 3x$

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations

Obtain answers $x=-2$ and $x=\frac{2}{5}$

OR: Obtain answer $x=-2$ by inspection or by solving a linear equation

Obtain answer $x=\frac{2}{5}$ similarly

(ii) Use correct method for solving an equation of the form $5^x=a$ or $5^{x+1}=a$, where $a>0$

Obtain answer $x=-0.569$ only

9709/31/M/J/16

Question 78:

Solve the inequality $2|x-2|>|3x+1|$. [4]

Answers:

EITHER: State or imply non-modular inequality $(2(x-2))^2>(3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2)=\pm(3x+1)$	**B1**
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
Obtain critical values $x=-5$ and $x=\frac{3}{5}$	**A1**
State final answer $-5<x<\frac{3}{5}$	**A1**
OR: Obtain critical value $x=-5$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
Obtain critical value $x=\frac{3}{5}$ similarly	**B2**
State final answer $-5<x<\frac{3}{5}$	**B1)**
[Do not condone $\le$ for $<$.]	[4]

9709/33/M/J/16

Question 79:

(i) Express x^2+6x+2 in the form $(x+a)^2+b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2+6x+2>9$. [2]

Answers:

(i)	$(x+3)^2-7$	B1B1	[2]	For $a=3$, $b=-7$
(ii)	$1,-7$ seen $x>1$, $x<-7$ oe	B1 B1	[2]	$x>1$ or $x<-7$ Allow $x\leqslant-7, x\geqslant 1$ oe

9709/11/O/N/16

Question 80:

Solve the equation $\dfrac{3^x+2}{3^x-2}=8$, giving your answer correct to 3 decimal places. [3]

Answers:

Solve for 3^x and obtain $3^x=\frac{18}{7}$	B1	
Use correct method for solving an equation of the form $3^x=a$, where $a>0$	M1	
Obtain answer $x=0.860$ 3 d.p. only	A1	[3]

9709/31/O/N/16

Question 81:

Solve the inequality $|x-3|<3x-4$. [4]

Answers:

EITHER: State or imply non-modular inequality $(x-3)^2 < (3x-4)^2$, or corresponding equation	(B1
Make reasonable attempt at solving a three term quadratic	M1
Obtain critical value $x = \frac{7}{4}$	A1
State final answer $x > \frac{7}{4}$ only	A1)
*OR*1: State the relevant critical inequality $3 - x < 3x - 4$, or corresponding equation	(B1
Solve for x	M1
Obtain critical value $x = \frac{7}{4}$	A1
State final answer $x > \frac{7}{4}$ only	A1)
*OR*2: Make recognizable sketches of $y = \lvert x-3 \rvert$ and $y = 3x - 4$ on a single diagram	(B1
Find x-coordinate of the intersection	M1
Obtain $x = \frac{7}{4}$	A1
State final answer $x > \frac{7}{4}$ only	A1)
Total:	**4**

9709/32/M/J/17

Question 82:

Using the substitution $u = e^x$, solve the equation $4e^{-x} = 3e^x + 4$. Give your answer correct to 3 significant figures. [4]

Answer:

Rearrange as $3u^2 + 4u - 4 = 0$, or $3e^{2x} + 4e^x - 4 = 0$, or equivalent	B1
Solve a 3-term quadratic for e^x or for u	M1
Obtain $e^x = \frac{2}{3}$ or $u = \frac{2}{3}$	A1
Obtain answer $x = -0.405$ and no other	A1

9709/33/M/J/17

Question 82:

It is given that the variable x is such that

$$1.3^{2x} < 80 \quad \text{and} \quad |3x - 1| > |3x - 10|.$$

Find the set of possible values of x, giving your answer in the form $a < x < b$ where the constants a and b are correct to 3 significant figures. [7]

Answers:

Take logarithms of both sides and apply power law	M1	Condone incorrect inequality signs until final answer. The first 6 marks are for obtaining the correct critical values.
Obtain $2x < \frac{\ln 80}{\ln 1.3}$ or equivalent using $\log_{10}$	A1	
Obtain $x = 8.35...$	A1	
State or imply non-modulus inequality $(3x-1)^2 > (3x-10)^2$ or corresponding equation or linear equation $3x-1 = -(3x-10)$	B1	
Attempt solution of inequality or equation (obtaining 3 terms when squaring each bracket or solving linear equation with signs of $3x$ different)	M1	
Obtain $x = \frac{11}{6}$ or $x = 1.83...$	A1	
Conclude $1.83 < x < 8.35$	A1	

9709/21/O/N/17

Question 83:

Solve the inequality $|3x - 2| < |x + 5|$. [4]

Answer:

Either	
State or imply non-modular inequality $(3x-2)^2 < (x+5)^2$ or corresponding equation or pair of linear equations	B1
Attempt solution of 3-term quadratic equation or of 2 linear equations	M1
Obtain critical values $-\frac{3}{4}$ and $\frac{7}{2}$	A1
State answer $-\frac{3}{4} < x < \frac{7}{2}$	A1
Or	
Obtain critical value $\frac{7}{2}$ from graph, inspection, equation	B1
Obtain critical value $-\frac{3}{4}$ similarly	B2
State answer $-\frac{3}{4} < x < \frac{7}{2}$	B1

9709/22/M/J/18

Question 84:

Showing all necessary working, solve the equation $3|2^x - 1| = 2^x$, giving your answers correct to 3 significant figures. [4]

Answers:

EITHER:	State or imply non-modular equation $3^2(2^x-1)^2=(2^x)^2$, or pair of equations $3(2^x-1)=\pm 2^x$	M1	$8(2^x)^2-18(2^x)+9=0$
	Obtain $2^x=\frac{3}{2}$ and $2^x=\frac{3}{4}$ or equivalent	A1	
OR:	Obtain $2^x=\frac{3}{2}$ by solving an equation	B1	
	Obtain $2^x=\frac{3}{4}$ by solving an equation	B1	
Use correct method for solving an equation of the form $2^x=a$, where $a>0$		M1	
Obtain **final** answers $x=0.585$ and $x=-0.415$ only		A1	The question requires 3 s.f. Do not ISW if they go on to reject one value

9709/32/M/J/18

Question 85:

Showing all necessary working, solve the equation $5^{2x}=5^x+5$. Give your answer correct to 3 decimal places. [5]

Answer:

State or imply $u^2=u+5$, or equivalent in 5^x	B1
Solve for u, or 5^x	M1
Obtain root $\frac{1}{2}(1+\sqrt{21})$, or decimal in [2.79, 2.80]	A1
Use correct method for finding x from a positive root	M1
Obtain answer $x=0.638$ and no other answer	A1

9709/33/M/J/18

Question 86:

Solve the inequality $|3x-5|<2|x|$. [4]

Answers:

Either		
State or imply non-modular inequality $(3x-5)^2 < 4x^2$ or corresponding equation or pair of linear equations	B1	SC: Common error $(3x-5)^2 < 2x^2$
Attempt solution of 3-term quadratic equation or solution of 2 linear equations	M1	
Obtain critical values 1 and 5	A1	Critical values $\frac{15 \pm 5\sqrt{2}}{7}$ or 3.15, 1.13 allow B1
State correct answer $1 < x < 5$	A1	$\frac{15-5\sqrt{2}}{7} < x < \frac{15+5\sqrt{2}}{7}$ or $1.13 < x < 3.15$ B1 Max 2/4 Allow M1 for $(7x \pm 5)(x \pm 5)$
Or		
Obtain $x = 5$ by solving linear equation or inequality or from graphical method or inspection	B1	Allow B1 for 5 seen, maybe in an inequality
Obtain $x = 1$ similarly	B2	Allow B2 for 1 seen, maybe in an inequality
State correct answer $1 < x < 5$	B1	

9709/22/O/N/18

Question 87:

Find the set of values of x satisfying the inequality $2|2x - a| < |x + 3a|$, where a is a positive constant. [4]

Answers:

EITHER:	State or imply non-modular inequality $2^2(2x-a)^2 < (x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a) = \pm(x+3a)$	B1
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1
	Obtain critical values $x = \frac{5}{3}a$ and $x = -\frac{1}{5}a$	A1
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$	A1
OR:	Obtain critical value $x = \frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1
	Obtain critical value $x = -\frac{1}{5}a$ similarly	B2
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	B1

9709/31/O/N/18

Question 88:

Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

Answer:

Rearrange the equation in the form $ae^{2x} = b$ or $ae^{x} = be^{-x}$	M1
Obtain correct equation in either form with $a = 2$ and $b = 5$	A1
Use correct method to solve for x	M1
Obtain answer $x = 0.46$	A1
	4

9709/31/O/N/18

Question 89:

(i) Solve the inequality $|3x - 5| < |x + 3|$. [4]

(ii) Hence find the greatest integer n satisfying the inequality $|3^{0.1n+1} - 5| < |3^{0.1n} + 3|$. [2]

Answers:

(i)	State or imply non-modular inequality $(3x-5)^2 < (x+3)^2$ or corresponding equation or pair of different linear equations/inequalities	B1	SC: Allow B1 for $x < 4$ from only one linear inequality
	Attempt solution of 3-term quadratic equation/inequality or of two different linear equations/inequalities	M1	For M1, must get as far as 2 critical values
	Obtain critical values $\frac{1}{2}$ and 4	A1	
	State answer $\frac{1}{2} < x < 4$ or equivalent	A1	If given as 2 separate statements, condone omission of 'and' or $\cap$ but penalise inclusion of 'or' or $\cup$
(ii)	Attempt to find n (not necessarily an integer so far) from $3^{0.1n} =$ or $<$ *their* positive upper value from part (i) or $3^{0.1n+1} =$ or $< 3 \times$ *their* positive upper value from part (i)	M1	0/2 for trial and improvement
	Conclude 12	A1	

9709/21/M/J/19

Question 90:

(i) Solve the equation $|4 + 2x| = |3 - 5x|$. [3]

(ii) Hence solve the equation $|4 + 2e^{3y}| = |3 - 5e^{3y}|$, giving the answer correct to 3 significant figures. [2]

Answers:

(i)	State or imply non-modular equation $(4+2x)^2 = (3-5x)^2$ or pair of linear equations	B1	
	Attempt solution of 3-term quadratic eqn or pair of linear equations	M1	
	Obtain $-\frac{1}{7}$, $\frac{7}{3}$	A1	SC B1 for $x = -\frac{1}{7}$ from one linear equation
		3	
(ii)	Attempt correct process to solve $e^{3y} = k$ where $k > 0$ from (i)	M1	
	Obtain 0.282 and no others	A1	

9709/22/M/J/19

Question 91:

Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

Answer:

State or imply $u^2 - u - 12(=0)$, or equivalent in 3^x	B1	Need to be convinced they know $3^{2x} = (3^x)^2$
Solve for u, or for 3^x, and obtain root 4	B1	
Use a correct method to solve an equation of the form $3^x = a$ where a >0	M1	Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x \ln 3 = \ln a$, $x = \log_3 a$ If seen, accept solution of straight forward cases such as $3^x = 3$, $x = 1$ without working
Obtain final answer $x = 1.26$ **only**	A1	The Q asks for 2 dp

9709/32/M/J/19

Question 92:

(i) Solve the inequality $|2x - 7| < |2x - 9|$. [3]

(ii) Hence find the largest integer n satisfying the inequality $|2 \ln n - 7| < |2 \ln n - 9|$. [2]

Answer:

(i)	State or imply non-modular inequality $(2x-7)^2 < (2x-9)^2$ or corresponding equation or linear equation (with signs of $2x$ different)
	Obtain critical value 4
	State $x < 4$ only
(ii)	Attempt to find n from $\ln n$ = *their* critical value from part **(i)**
	Obtain or imply $n < e^4$ and hence 54

9709/21/O/N/19

Question 93:

(i) Solve the equation $|4x + 5| = |x - 7|$. [3]

(ii) Hence, using logarithms, solve the equation $|2^{y+2} + 5| = |2^y - 7|$, giving the answer correct to 3 significant figures. [2]

Answer:

(i)	State or imply non-modular equation $(4x+5)^2 = (x-7)^2$ or pair of different linear equations	B1	
	Attempt solution of 3-term quadratic equation or pair of linear equations	M1	
	Obtain $\frac{2}{5}$ and -4	A1	SC For $x = -4$ only, from correct work, allow B1
(ii)	Apply logarithms and use power law for $2^y = k$ where $k > 0$ from (i)	M1	
	Obtain -1.32 only	A1	AWRT

9709/22/O/N/19

Question 94:

Solve the inequality $|2x - 3| > 4|x + 1|$. [4]

Answer:

State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3)=\pm4(x+1)$	B1	$12x^2+44x+7<0$
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	Correct method seen, or implied by correct answers
Obtain critical values $x=-\frac{7}{2}$ and $x=-\frac{1}{6}$	A1	
State final answer $-\frac{7}{2}<x<-\frac{1}{6}$	A1	
Alternative method for question 2		
Obtain critical value $x=-\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
Obtain critical value $x=-\frac{1}{6}$ similarly	B2	
State final answer $-\frac{7}{2}<x<-\frac{1}{6}$	B1	

9709/31/O/N/19

Question 95:

Solve the inequality $2|x+2| > |3x-1|$. [4]

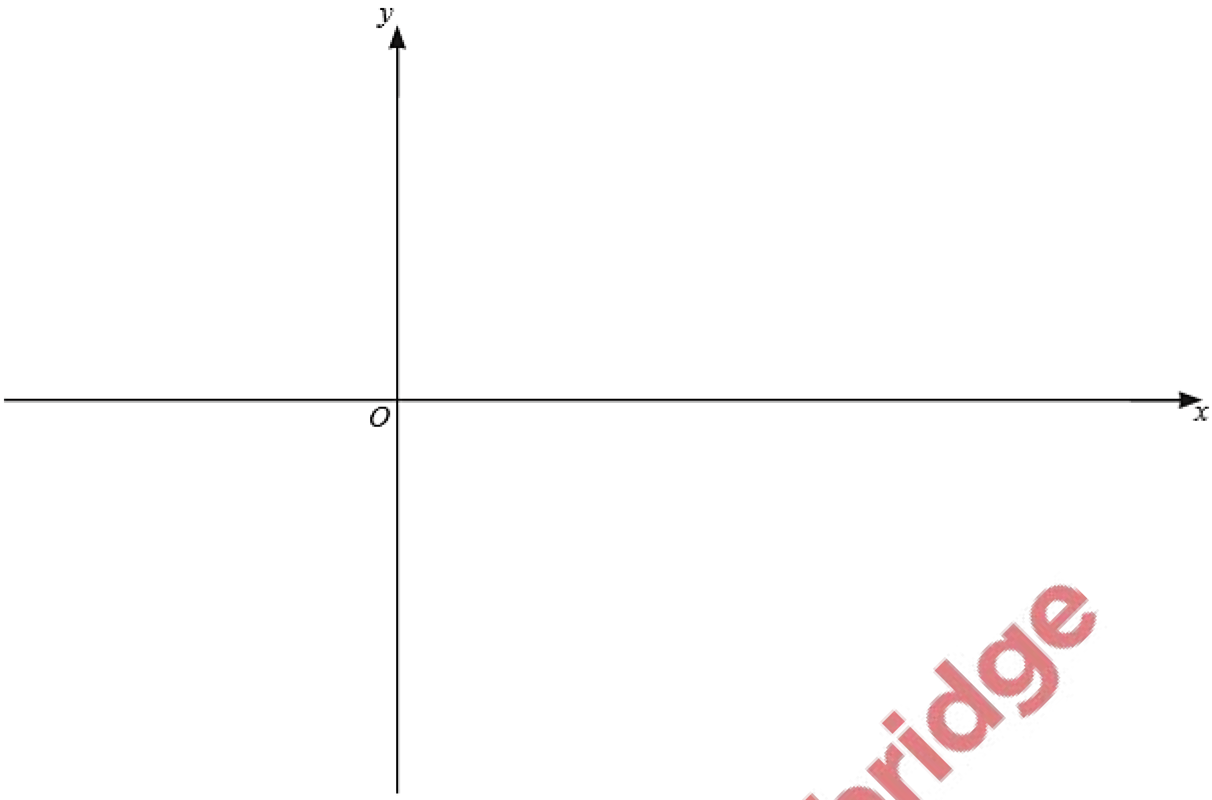
Answer:

State or imply non-modular inequality $(x+2)^2>(3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x+2)=\pm(3x-1)$
Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x
Obtain critical values $x=-\frac{3}{5}$ and $x=5$
State final answer $-\frac{3}{5}<x<5$
Alternative method for question 1
Obtain critical value $x=5$ from a graphical method, or by inspection, or by solving a linear equation or an inequality
Obtain critical value $x=-\frac{3}{5}$ similarly
State final answer $-\frac{3}{5}<x<5$

9709/33/O/N/19

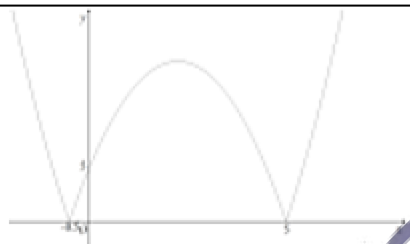
Question 96:

(i) On the axes below, sketch the graph of $y=|2x^2-9x-5|$ showing the coordinates of the points where the graph meets the axes. [4]



(ii) Find the values of k for which $|2x^2 - 9x - 5| = k$ has exactly 2 solutions. [3]

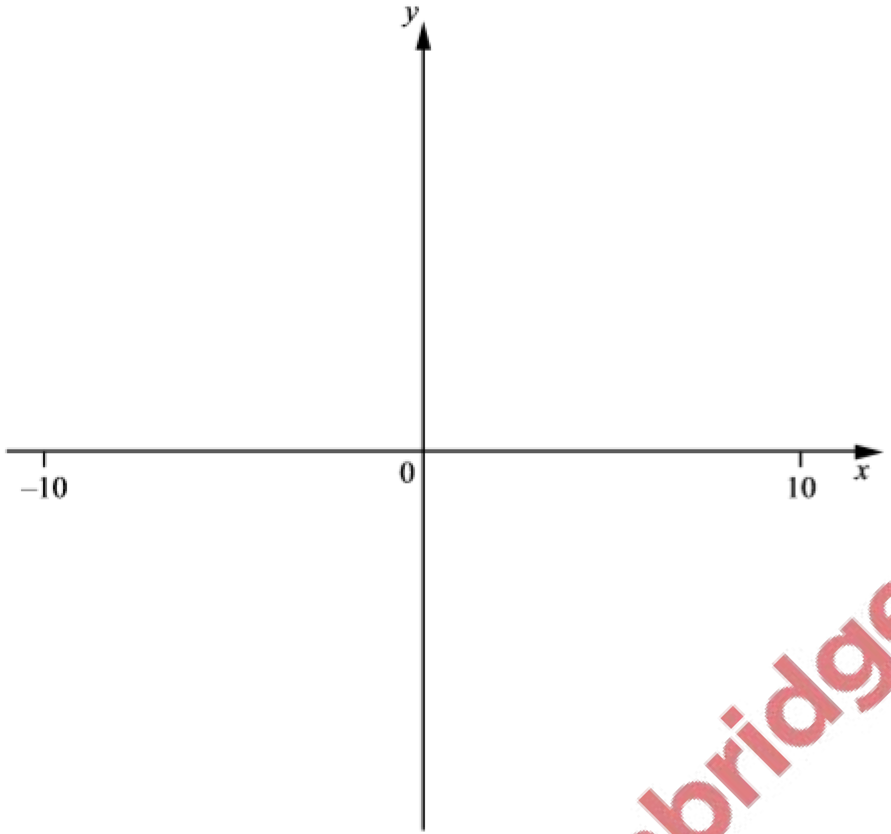
Answer:

4(i)		**B4**	**B1** for shape, with max in first quadrant **B1** for $(-0.5, 0)$ and $(5, 0)$ **B1** for $(0, 5)$ **B1** all correct, with cusps and correct curvature for $x < 0.5$ and $x > 5$
4(ii)	$k = 0$	**B1**	Not from incorrect work
	Stationary point when $y = \pm\frac{121}{8}$ or ± 15.125	**M1**	For attempt to find y-coordinate of stationary point, must be a complete method i.e. Use of calculus Use of discriminant, Use of completing the square Use of symmetry Allow if seen in part (i), but must be used in (ii)
	$k > \frac{121}{8}$	**A1**	cao

4037/12/O/N/19

Question 97:

(a) On the axes below, sketch the graph of $y = |2x + 5|$ and the graph of $y = |2 - x|$, stating the coordinates of the points where each graph meets the coordinate axes. [4]



(b) Solve $|2x+5| \leqslant |2-x|$. [3]

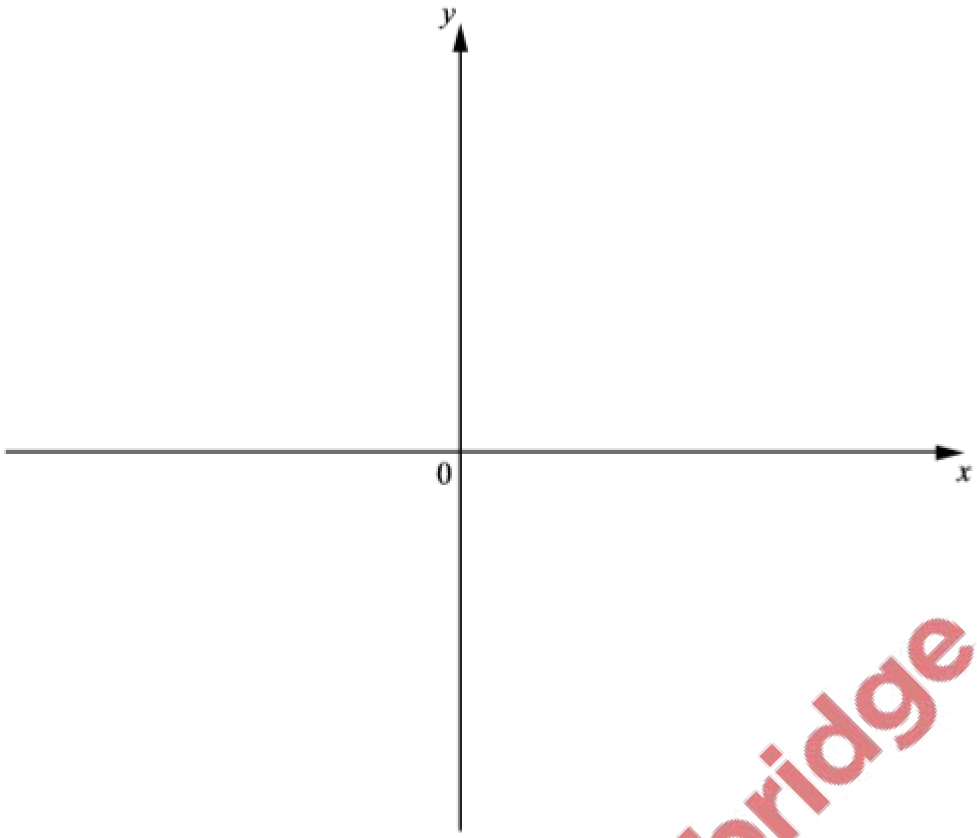
Answers:

(a)	Correct shape for both graphs	B2	**B1** for either Must touch the x-axis in the correct quadrant
	Correct y-intercept for both graphs and Correct x-intercept for both graphs	B2	**B1** for either $y=2,\ y=5$ or $x=2,\ x=-2.5$ or $y=2,\ x=-2.5$
(b)	$2x+5=\pm(2-x)$ oe or $(2x+5)^2=(2-x)^2$	M1	For attempt to obtain 2 solutions; must be a complete method
	$x=-7, x=-1$	A1	
	$-7 \leqslant x \leqslant -1$	A1	**FT** *their* values of x

4037/01/SP/20

Question 98:

(a) On the axes below, sketch the graph of $y=\frac{1}{5}(x-2)(x-4)(x+5)$, showing the coordinates of the points where the graph meets the coordinate axes.

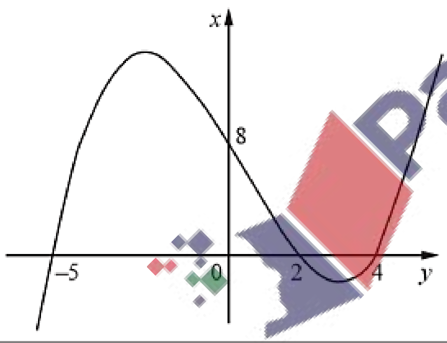


[2]

(b) Explain why your sketch in part **(a)** can be used to solve $(x-2)(x-4)(x+5) \leqslant 0$. [1]

(c) Hence solve $(x-2)(x-4)(x+5) \leqslant 0$. [1]

Answers:

(a)		2	**B1** for shape **B1** for intercepts on coordinate axes
(b)	Valid explanation, e.g. multiplying throughout by 5 does not change x values because the x-axis is invariant.	1	
(c)	$x \leqslant -5$ $2 \leqslant x \leqslant 4$	1	**FT** *their* graph

4037/02/SP/20